Quasi-Variational Inequalities, Generalized Nash Equilibria, and Multi-Leader-Follower Games: Erratum

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Abstract. In [1], a sequential penalty approach was presented for a quasi-variational inequality (QVI) with particular application to the generalized Nash game. To test the computational performance of the penalty method, numerical results were reported with an example from a multi-leader-follower game in an electric power market. However, due to an inverted sign in the penalty term in the example and some missing terms in the derivatives of the firms' Lagrangian functions, the reported numerical results in [1] are incorrect. Since the numerical examples of this kind are scarce in the literature and this particular example may be useful in the future research, we report the corrected results.

In Subsection 5.3 of [1], to test the computational performance of the proposed penalty method for a quasi-variational inequality (QVI), numerical experiments were carried out on an example from a multi-leader-follower game in an electric power market. However, due to an inverted sign in the penalty term in the example and some missing terms in the derivatives of the firms' Lagrangian functions, the reported numerical results in [1] are incorrect.

First, on page 52, firm f's penalized problem should be written as

$$\begin{array}{ll} \text{maximize} & \theta_f(s^{-f},\ s^f, w) - \dfrac{1}{2\rho_k} \displaystyle\sum_{i,j\in\mathcal{N}} \max(0,\ u_{ij}^{k,f} + \rho_k(p_j(S_j) - p_i(S_i) - w_{ij}))^2 \\ \text{subject to} & \displaystyle\sum_{j\in\mathcal{N}} s_{ij}^f \leq \text{CAP}_i^f, \qquad \forall i \in \mathcal{N} \\ & s_{ij}^f \geq 0, \qquad \forall (i,j) \in \mathcal{N}\times\mathcal{N}, \end{array}$$

where

$$\theta_f(s^{-f},\ s^f, w) = \sum_{j\in\mathcal{N}} \left[p_j(S_j) \sum_{i\in\mathcal{N}} s_{ij}^f \right] - \sum_{i,j\in\mathcal{N}} w_{ij} s_{ij}^f - \sum_{i\in\mathcal{N}} \left(c_i^f \sum_{j\in\mathcal{N}} s_{ij}^f \right).$$

Next, the LCP shown on the same page should be written as follows: For all $f \in \mathcal{F}$ and $(i,\ j) \in \mathcal{N}\times\mathcal{N}$,

$$0 \ \leq s_{ij}^f \ \perp\ c_i^f + e_{ij} - v_{ij} - p_j(S_j) + \frac{P_j^0}{Q_j^0} \sum_{l\in\mathcal{N}} s_{lj}^f + \mu_i^f - \frac{P_j^0}{Q_j^0} \left(\sum_{l\in\mathcal{N}} \lambda_{lj}^{k,f} - \sum_{m\in\mathcal{N}} \lambda_{jm}^{k,f} \right) \geq 0$$

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Table 1: Firms' sales and nodal prices. (cf. Tables 2 and 3 in [1])

node i	node j	firm	sales s_{ij}^f	price p_i
1				28.74
2				28.07
3				27.74
1	1	1	43.54	
1	2	1	28.14	
1	3	1	28.33	
2	1	1	26.87	
2	2	1	11.47	
2	3	1	11.66	
2	1	2	43.54	
2	2	2	28.14	
2	3	2	28.33	
3	1	2	26.87	
3	2	2	11.47	
3	3	2	11.66	

$$
\begin{aligned}
0 &\leq \mu_i^f \;\perp\; \mathrm{CAP}_i^f - \sum_{l \in \mathcal{N}} s_{il}^f \geq 0 \\
0 &\leq v_{ij} \;\perp\; \sum_{t \in \mathcal{F}} s_{ij}^t \geq 0 \\
0 &\leq \lambda_{ij}^{k,f} \;\perp\; \lambda_{ij}^{k,f} - u_{ij}^{k,f} - \rho_k \left(p_j(S_j) - p_i(S_i) - e_{ij} + v_{ij}\right) \geq 0
\end{aligned}
$$

This example was solved with the same data as those shown in Table 1 of [1] and the same settings as those described on page 53 of [1]. After 7 iterations, the termination criterion was satisfied. The obtained results are shown in the table. The ISO's shipping charges w_{ij} were all equal to e_{ij}. Moreover, the same problem was solved with the alternative penalty update rule $\rho_{k+1} := 2\rho_k$. Then the same solution was obtained after 19 iterations.

Finally, as in [1], firm II's costs c_i^{II} were changed to 20 at all 3 nodes. Then the problem was successfully solved with either of the penalty update rules $\rho_{k+1} := 10\rho_k$ and $\rho_{k+1} := 2\rho_k$. The two update rules produced the same solution satisfying the optimality conditions for the original problem.

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References

[1] Pang JS, Fukushima M (2005) Quasi-variational inequalities, generalized Nash equilibria, and multi-leader-follower games. *Computational Management Science* **2**: 21–56.