A Modified Relaxation Scheme for Mathematical Programs with Complementarity Constraints: Erratum

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In [1], a modified relaxation method was proposed for mathematical programs with complementarity constraints and some new sufficient conditions for M- or B-stationarity were shown. However, due to an ignored sign in the Lagrangian function of the relaxed problem, the proofs of Theorems 3.4 and 3.5 in [1] are incorrect. In what follows, we give the corrected proofs. Throughout, we use the same notations as in [1].

Theorem 3.4. *Let* $\{\epsilon_k\} \subseteq (0, +\infty)$ *be convergent to* 0 *and* $z^k \in \mathcal{F}_{\epsilon_k}$ *be a stationary point of problem (3) with* $\epsilon = \epsilon_k$ *and multiplier vectors* $\lambda^k, \mu^k, \delta^k$*, and* γ^k*. Suppose that, for each* k*,* $\nabla_z^2 L_{\epsilon_k}(z^k, \lambda^k, \mu^k, \delta^k, \gamma^k)$ *is bounded below with constant* α_k *on the corresponding tangent space* $\mathcal{T}_{\epsilon_k}(z^k)$*. Let* $\bar{z}$ *be an accumulation point of the sequence* $\{z^k\}$*. If the sequence* $\{\alpha_k\}$ *is bounded and the MPEC-LICQ holds at* $\bar{z}$*, then* $\bar{z}$ *is an M-stationary point of problem (1).*

Proof. Assume that $\lim_{k\to\infty} z^k = \bar{z}$ without loss of generality. First of all, we note from Theorem 3.3 that $\bar{z}$ is a C-stationary point of problem (1). To prove the theorem, we assume to the contrary that $\bar{z}$ is not M-stationary to problem (1). Then, it follows from the definitions of C-stationarity and M-stationarity that there must exist an $i_0 \in \mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})$ such that

$$\bar{u}_{i_0} < 0, \quad \bar{v}_{i_0} < 0. \tag{49}$$

By (39)–(40) and (45)–(46), we have

$$i_0 \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k)$$

for every sufficiently large k. We first claim that $i_0 \notin \mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ for all k sufficiently large. In fact, if there exists a subsequence $\{z^k\}_{k\in\mathcal{K}}$ such that $i_0 \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ for all $k \in \mathcal{K}$, then, by (39) and (40), we have from (49) that

$$\begin{aligned} \bar{u}_{i_0} &= \lim_{\substack{k\in\mathcal{K} \\ k\to\infty}} \delta_{i_0}^k (H_{i_0}(z^k) + \epsilon_k) < 0, \\ \bar{v}_{i_0} &= \lim_{\substack{k\in\mathcal{K} \\ k\to\infty}} \delta_{i_0}^k (G_{i_0}(z^k) + \epsilon_k) < 0. \end{aligned}$$

Since $\delta_{i_0}^k \geq 0$ for each k, when $k \in \mathcal{K}$ is sufficiently large, there hold

$$H_{i_0}(z^k) < -\epsilon_k, \qquad G_{i_0}(z^k) < -\epsilon_k$$

and hence $H_{i_0}(z^k) G_{i_0}(z^k) > \epsilon_k^2$. This contradicts the fact that, for each k, z^k is a feasible point of problem (3) with $\epsilon = \epsilon_k$. Therefore, we have $i_0 \notin \mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ for all sufficiently large k, which implies

$$i_0 \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \tag{50}$$

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for all sufficiently large k. Then, by (39) and (40),

$$\bar{u}_{i_0} = -\lim_{k\to\infty} \gamma_{i_0}^k H_{i_0}(z^k) < 0, \tag{51}$$

$$\bar{v}_{i_0} = -\lim_{k\to\infty} \gamma_{i_0}^k G_{i_0}(z^k) < 0, \tag{52}$$

and so

$$\lim_{k\to\infty} \frac{H_{i_0}(z^k)}{G_{i_0}(z^k)} = \frac{\bar{u}_{i_0}}{\bar{v}_{i_0}} > 0. \tag{53}$$

In what follows, we suppose that, for all sufficiently large k, (28)–(31), (35), and

$$\frac{H_{i_0}(z^k)}{G_{i_0}(z^k)} > 0$$

hold and all the matrix functions $A_i(z,\epsilon)$, $i = 1, \cdots, N$, in (7) have full column rank at (z^k, ϵ_k). For such k, the matrix $A_{N_k}(z^k, \epsilon_k)$ whose columns consist of the vectors

$$\begin{aligned}
&\nabla g_l(z^k): \quad l \in \mathcal{I}_g(\bar{z}),\\
&\nabla h_r(z^k): \quad r = 1, \cdots, q,\\
&\nabla G_i(z^k): \quad i \in \Big(\mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})\Big) \cup \Big(\mathcal{I}_G(\bar{z}) \setminus (\mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k))\Big),\\
&\nabla G_i(z^k) + \frac{G_i(z^k)+\epsilon_k}{H_i(z^k)+\epsilon_k}\nabla H_i(z^k): \quad i \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_H(\bar{z}),\\
&\nabla G_i(z^k) + \frac{G_i(z^k)}{H_i(z^k)}\nabla H_i(z^k): \qquad i \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_H(\bar{z}),\\
&\nabla H_j(z^k): \quad j \in \Big(\mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})\Big) \cup \Big(\mathcal{I}_H(\bar{z}) \setminus (\mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k))\Big),\\
&\nabla H_j(z^k) + \frac{H_j(z^k)+\epsilon_k}{G_j(z^k)+\epsilon_k}\nabla G_j(z^k): \quad j \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_G(\bar{z}),\\
&\nabla H_j(z^k) + \frac{H_j(z^k)}{G_j(z^k)}\nabla G_j(z^k): \qquad j \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_G(\bar{z})
\end{aligned}$$

has full column rank. Therefore, we can choose a vector $d^k \in R^n$ such that

$$(d^k)^T \nabla g_l(z^k) = 0, \qquad l \in \mathcal{I}_g(\bar{z}); \tag{54}$$

$$(d^k)^T \nabla h_r(z^k) = 0, \qquad r = 1, \cdots, q; \tag{55}$$

$$(d^k)^T \nabla G_i(z^k) = 0, \quad i \in \Big(\mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})\Big) \cup \Big(\mathcal{I}_G(\bar{z}) \setminus (\mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k))\Big), \ i \neq i_0; \tag{56}$$

$$(d^k)^T \Big(\nabla G_i(z^k) + \frac{G_i(z^k)+\epsilon_k}{H_i(z^k)+\epsilon_k}\nabla H_i(z^k)\Big) = 0, \quad i \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_H(\bar{z}); \tag{57}$$

$$(d^k)^T \Big(\nabla G_i(z^k) + \frac{G_i(z^k)}{H_i(z^k)}\nabla H_i(z^k)\Big) = 0, \qquad i \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_H(\bar{z}); \tag{58}$$

$$(d^k)^T \nabla H_j(z^k) = 0, \quad j \in \Big(\mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})\Big) \cup \Big(\mathcal{I}_H(\bar{z}) \setminus (\mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k))\Big), \ j \neq i_0; \tag{59}$$

$$(d^k)^T \Big(\nabla H_j(z^k) + \frac{H_j(z^k)+\epsilon_k}{G_j(z^k)+\epsilon_k}\nabla G_j(z^k)\Big) = 0, \quad j \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_G(\bar{z}); \tag{60}$$

$$(d^k)^T \Big(\nabla H_j(z^k) + \frac{H_j(z^k)}{G_j(z^k)}\nabla G_j(z^k)\Big) = 0, \qquad j \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_G(\bar{z}); \tag{61}$$

$$(d^k)^T \nabla G_{i_0}(z^k) = 1; \tag{62}$$

$$(d^k)^T \nabla H_{i_0}(z^k) = -\frac{H_{i_0}(z^k)}{G_{i_0}(z^k)}.$$

Then for any $i \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ and any $j \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k)$, since

$$\begin{aligned}
\nabla \phi_{\epsilon_k,i}(z^k) &= (G_i(z^k)+\epsilon_k)\nabla H_i(z^k) + (H_i(z^k)+\epsilon_k)\nabla G_i(z^k),\\
\nabla \psi_{\epsilon_k,j}(z^k) &= H_j(z^k)\nabla G_j(z^k) + G_j(z^k)\nabla H_j(z^k),
\end{aligned}$$

we have

$$\begin{aligned}
(d^k)^T\nabla \phi_{\epsilon_k,i}(z^k) = 0, \quad i \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k),\\
(d^k)^T\nabla \psi_{\epsilon_k,j}(z^k) = 0, \quad j \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k),
\end{aligned}$$

and so $d^k \in \mathcal{T}_{\epsilon_k}(z^k)$. Furthermore, we can choose the sequence $\{d^k\}$ to be bounded. Since $\nabla_z^2 L_{\epsilon_k}(z^k,\lambda^k,\ \mu^k,\delta^k,\gamma^k)$ is bounded below with constant α_k on the corresponding tangent space $\mathcal{T}_{\epsilon_k}(z^k)$, we have from (48) that there exists a constant C such that

$$(d^k)^T\nabla_z^2 L_{\epsilon_k}(z^k,\lambda^k,\mu^k,\delta^k,\gamma^k)d^k \geq -\alpha_k||d^k||^2 \geq C, \tag{63}$$

where the last inequality follows from the boundedness of the sequences $\{\alpha_k\}$ and $\{d^k\}$. Note that, by (32)–(34) and

$$\begin{aligned}
\nabla^2 \phi_{\epsilon_k,i}(z^k) &= \nabla G_i(z^k)\nabla H_i(z^k)^T + \nabla H_i(z^k)\nabla G_i(z^k)^T\\
&\quad +(G_i(z^k)+\epsilon_k)\nabla^2 H_i(z^k) + (H_i(z^k)+\epsilon_k)\nabla^2 G_i(z^k),\\
\nabla^2 \psi_{\epsilon_k,j}(z^k) &= \nabla G_j(z^k)\nabla H_j(z^k)^T + \nabla H_j(z^k)\nabla G_j(z^k)^T\\
&\quad +G_j(z^k)\nabla^2 H_j(z^k) + H_j(z^k)\nabla^2 G_j(z^k),
\end{aligned}$$

there holds

$$\begin{aligned}
\nabla_z^2 L_{\epsilon_k}(z^k,\lambda^k,\mu^k,\delta^k,\gamma^k) &= \nabla^2 f(z^k) + \sum_{l=1}^{p}\lambda_l^k\nabla^2 g_l(z^k) + \sum_{r=1}^{q}\mu_r^k\nabla^2 h_r(z^k)\\
&\quad -\sum_{i=1}^{m}\delta_i^k\nabla^2\phi_{\epsilon_k,i}(z^k) + \sum_{j=1}^{m}\gamma_j^k\nabla^2\psi_{\epsilon_k,j}(z^k)\\
&= \nabla^2 f(z^k) + \sum_{l\in\mathcal{I}_g(\bar{z})}\lambda_l^k\nabla^2 g_l(z^k) + \sum_{r=1}^{q}\mu_r^k\nabla^2 h_r(z^k)\\
&\quad -\sum_{i\in\mathcal{I}_{\Phi_{\epsilon_k}}(z^k)}\delta_i^k\nabla^2\phi_{\epsilon_k,i}(z^k) + \sum_{j\in\mathcal{I}_{\Psi_{\epsilon_k}}(z^k)}\gamma_j^k\nabla^2\psi_{\epsilon_k,j}(z^k).
\end{aligned}$$

We then have

$$\begin{aligned}
&(d^k)^T\nabla_z^2 L_{\epsilon_k}(z^k,\lambda^k,\mu^k,\delta^k,\gamma^k)d^k\\
&= (d^k)^T\nabla^2 f(z^k)d^k + \sum_{l\in\mathcal{I}_g(\bar{z})}\lambda_l^k(d^k)^T\nabla^2 g_l(z^k)d^k + \sum_{r=1}^{q}\mu_r^k(d^k)^T\nabla^2 h_r(z^k)d^k\\
&\quad -\sum_{i\in\mathcal{I}_{\Phi_{\epsilon_k}}(z^k)}\delta_i^k\Big((d^k)^T\nabla G_i(z^k)\nabla H_i(z^k)^Td^k + (d^k)^T\nabla H_i(z^k)\nabla G_i(z^k)^Td^k\\
&\qquad + (G_i(z^k)+\epsilon_k)(d^k)^T\nabla^2 H_i(z^k)d^k + (H_i(z^k)+\epsilon_k)(d^k)^T\nabla^2 G_i(z^k)d^k\Big)\\
&\quad +\sum_{j\in\mathcal{I}_{\Psi_{\epsilon_k}}(z^k)}\gamma_j^k\Big((d^k)^T\nabla G_j(z^k)\nabla H_j(z^k)^Td^k + (d^k)^T\nabla H_j(z^k)\nabla G_j(z^k)^Td^k\\
&\qquad + G_j(z^k)(d^k)^T\nabla^2 H_j(z^k)d^k + H_j(z^k)(d^k)^T\nabla^2 G_j(z^k)d^k\Big).
\end{aligned} \tag{64}$$

By the twice continuous differentiability of the functions, the boundness of the sequence $\{d^k\}$, and the convergence of the sequences $\{z^k\}$, $\{\lambda_l^k\}$ and $\{\mu_r^k\}$ (by (43)–(44)), the terms

$$(d^k)^T\nabla^2 f(z^k)d^k, \quad \sum_{l\in\mathcal{I}_g(\bar z)} \lambda_l^k(d^k)^T\nabla^2 g_l(z^k)d^k, \quad \sum_{r=1}^{q}\mu_r^k(d^k)^T\nabla^2 h_r(z^k)d^k$$

are all bounded. Consider arbitrary indices i and j such that $i\in\mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ for infinitely many k and $j\in\mathcal{I}_{\Psi_{\epsilon_k}}(z^k)\setminus\{i_0\}$ for infinitely many k, respectively. If

$$i\in\mathcal{I}_G(\bar z)\cap\mathcal{I}_H(\bar z) \quad\text{or}\quad j\in\mathcal{I}_G(\bar z)\cap\mathcal{I}_H(\bar z),$$

then

$$(d^k)^T\nabla G_i(z^k)=0 \quad\text{or}\quad (d^k)^T\nabla H_j(z^k)=0$$

and, by (39)–(40) and (45)–(46), the sequences

$$\Big\{\delta_i^k(G_i(z^k)+\epsilon_k)\Big\}, \quad \Big\{\delta_i^k(H_i(z^k)+\epsilon_k)\Big\},$$

and

$$\Big\{\gamma_j^k G_j(z^k)\Big\}, \quad \Big\{\gamma_j^k H_j(z^k)\Big\}$$

are all convergent. If

$$i,j\notin\mathcal{I}_G(\bar z)\cap\mathcal{I}_H(\bar z),$$

then, also by (39)–(40) and (45)–(46), the sequences $\{\delta_i^k\}$ and $\{\gamma_j^k\}$ are convergent. Therefore, we have that the terms

$$\sum_{i\in\mathcal{I}_{\Phi_{\epsilon_k}}(z^k)} \delta_i^k\Big((d^k)^T\nabla G_i(z^k)\nabla H_i(z^k)^Td^k+(d^k)^T\nabla H_i(z^k)\nabla G_i(z^k)^Td^k+ \\ (G_i(z^k)+\epsilon_k)(d^k)^T\nabla^2 H_i(z^k)d^k+(H_i(z^k)+\epsilon_k)(d^k)^T\nabla^2 G_i(z^k)d^k\Big)$$

and

$$\sum_{j\in\mathcal{I}_{\Psi_{\epsilon_k}}(z^k)\setminus\{i_0\}} \gamma_j^k\Big((d^k)^T\nabla G_j(z^k)\nabla H_j(z^k)^Td^k+(d^k)^T\nabla H_j(z^k)\nabla G_j(z^k)^Td^k+ \\ G_j(z^k)(d^k)^T\nabla^2 H_j(z^k)d^k+H_j(z^k)(d^k)^T\nabla^2 G_j(z^k)d^k\Big)$$

are bounded. On the other hand, however, we have (50) for all sufficiently large k and

$$\begin{aligned} &\gamma_{i_0}^k\Big((d^k)^T\nabla G_{i_0}(z^k)\nabla H_{i_0}(z^k)^Td^k+(d^k)^T\nabla H_{i_0}(z^k)\nabla G_{i_0}(z^k)^Td^k \\ &+G_{i_0}(z^k)(d^k)^T\nabla^2 H_{i_0}(z^k)d^k+H_{i_0}(z^k)(d^k)^T\nabla^2 G_{i_0}(z^k)d^k\Big) \\ =\ &-\frac{2\gamma_{i_0}^k H_{i_0}(z^k)}{G_{i_0}(z^k)}+\gamma_{i_0}^k\Big(G_{i_0}(z^k)(d^k)^T\nabla^2 H_{i_0}(z^k)d^k+H_{i_0}(z^k)(d^k)^T\nabla^2 G_{i_0}(z^k)d^k\Big). \end{aligned} \tag{65}$$

Since (53) holds and $\gamma_{i_0}^k\to+\infty$ as $k\to\infty$ by (29) and (51), we have

$$-\frac{2\gamma_{i_0}^k H_{i_0}(z^k)}{G_{i_0}(z^k)}\to-\infty$$

as $k \to \infty$. Note that, by (51) and (52), the sequences

$$\left\{\gamma_{i_0}^k G_{i_0}(z^k)\right\}, \quad \left\{\gamma_{i_0}^k H_{i_0}(z^k)\right\}$$

are also convergent. We then have that the term (65) tends to $-\infty$ as $k \to \infty$. Therefore, it follows from (64) that

$$(d^k)^T \nabla_z^2 L_{\epsilon_k}(z^k, \lambda^k, \mu^k, \delta^k, \gamma^k) d^k \to -\infty$$

as $k \to \infty$. This contradicts (63) and hence $\bar{z}$ is M-stationary to problem (1). □

Theorem 3.5. *Let $\{\epsilon_k\}, \{z^k\}$, and $\bar{z}$ be the same as in Theorem 3.4 and $\lambda^k, \mu^k, \delta^k$, and γ^k be the multiplier vectors corresponding to z^k. Let β_k be the smallest eigenvalue of the matrix $\nabla_z^2 L_{\epsilon_k}(z^k, \lambda^k, \mu^k, \delta^k, \gamma^k)$. If the sequence $\{\beta_k\}$ is bounded below and the MPEC-LICQ holds at $\bar{z}$, then $\bar{z}$ is a B-stationary point of problem (1).*

Proof. It is easy to see that the assumptions of Theorem 3.4 are satisfied with $\alpha_k = \max\{-\beta_k, 0\}$ and so $\bar{z}$ is an M-stationary point of problem (1). Suppose that $\bar{z}$ is not B-stationary to problem (1). Then, by the definitions of B- and M-stationarity, there exists an $i_0 \in \mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})$ such that

$$\bar{u}_{i_0} < 0, \quad \bar{v}_{i_0} = 0 \tag{66}$$

or

$$\bar{u}_{i_0} = 0, \quad \bar{v}_{i_0} < 0.$$

Without loss of generality, we assume that (66) holds. By (39)–(40) and (45)–(46), we have

$$i_0 \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k)$$

for every sufficiently large k. If there exists a subsequence $\{z^k\}_{k\in\mathcal{K}}$ such that $i_0 \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ for all $k \in \mathcal{K}$, we have from (39), (45), and (66) that $\bar{u}_{i_0} = \lim_{k\in\mathcal{K},k\to\infty} \delta_{i_0}^k (H_{i_0}(z^k) + \epsilon_k) < 0$, which implies $H_{i_0}(z^k) + \epsilon_k < 0$ when $k \in \mathcal{K}$ is sufficiently large. Since $(H_{i_0}(z^k) + \epsilon_k)(G_{i_0}(z^k) + \epsilon_k) \geq \epsilon_k^2$ for each k, there also holds $G_{i_0}(z^k) + \epsilon_k < 0$ for all $k \in \mathcal{K}$ sufficiently large. Thus, there must hold $H_{i_0}(z^k)G_{i_0}(z^k) > \epsilon_k^2$ when $k \in \mathcal{K}$ is sufficiently large, which contradicts the fact that z^k is feasible to problem (3) with $\epsilon = \epsilon_k$ for each k. Therefore, we have $i_0 \notin \mathcal{I}_{\Phi_{\epsilon_k}}(z^k)$ for all sufficiently large k, which yields

$$i_0 \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \tag{67}$$

for all sufficiently large k. Then, it follows from (39), (40), and (66) that

$$\bar{u}_{i_0} = -\lim_{k\to\infty} \gamma_{i_0}^k H_{i_0}(z^k) < 0$$

and so, by (29), we have

$$\lim_{k\to\infty} \gamma_{i_0}^k = +\infty. \tag{68}$$

Now we suppose that, for all sufficiently large k, (28)–(31) and (35) hold and the matrix $A_{N_k}(z^k, \epsilon_k)$ defined in the proof of Theorem 3.4 has full column rank. Therefore, we can choose a vector $d^k \in R^n$

such that

$$
\begin{aligned}
&(d^k)^T \nabla g_l(z^k) = 0, \qquad\qquad l \in \mathcal{I}_g(\bar{z}); \\
&(d^k)^T \nabla h_r(z^k) = 0, \qquad\qquad r = 1, \cdots, q; \\
&(d^k)^T \nabla G_i(z^k) = 0, \quad i \in \Big(\mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})\Big) \cup \Big(\mathcal{I}_G(\bar{z}) \setminus (\mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k))\Big),\ i \neq i_0; \\
&(d^k)^T \Big(\nabla G_i(z^k) + \frac{G_i(z^k) + \epsilon_k}{H_i(z^k) + \epsilon_k} \nabla H_i(z^k)\Big) = 0, \quad i \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_H(\bar{z}); \\
&(d^k)^T \Big(\nabla G_i(z^k) + \frac{G_i(z^k)}{H_i(z^k)} \nabla H_i(z^k)\Big) = 0, \qquad i \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_H(\bar{z}); \\
&(d^k)^T \nabla H_j(z^k) = 0, \quad j \in \Big(\mathcal{I}_G(\bar{z}) \cap \mathcal{I}_H(\bar{z})\Big) \cup \Big(\mathcal{I}_H(\bar{z}) \setminus (\mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \cup \mathcal{I}_{\Psi_{\epsilon_k}}(z^k))\Big),\ j \neq i_0; \\
&(d^k)^T \Big(\nabla H_j(z^k) + \frac{H_j(z^k) + \epsilon_k}{G_j(z^k) + \epsilon_k} \nabla G_j(z^k)\Big) = 0, \quad j \in \mathcal{I}_{\Phi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_G(\bar{z}); \\
&(d^k)^T \Big(\nabla H_j(z^k) + \frac{H_j(z^k)}{G_j(z^k)} \nabla G_j(z^k)\Big) = 0, \qquad j \in \mathcal{I}_{\Psi_{\epsilon_k}}(z^k) \setminus \mathcal{I}_G(\bar{z}); \\
&(d^k)^T \nabla G_{i_0}(z^k) = 1; \\
&(d^k)^T \nabla H_{i_0}(z^k) = -1.
\end{aligned}
$$

Furthermore, we can choose the sequence $\{d^k\}$ to be bounded. By the assumptions of the theorem, there exists a constant C such that

$$
(d^k)^T \nabla_z^2 L_{\epsilon_k}(z^k, \lambda^k, \mu^k, \delta^k, \gamma^k) d^k \geq \beta_k ||d^k||^2 \geq C \tag{69}
$$

holds for all k. In a similar way to the proof of Theorem 3.4, we can show that all the terms on the right-hand side of (64) except

$$
\begin{aligned}
\gamma_{i_0}^k \Big(&(d^k)^T \nabla G_{i_0}(z^k) \nabla H_{i_0}(z^k)^T d^k + (d^k)^T \nabla H_{i_0}(z^k) \nabla G_{i_0}(z^k)^T d^k \\
&+ G_{i_0}(z^k)(d^k)^T \nabla^2 H_{i_0}(z^k) d^k + H_{i_0}(z^k)(d^k)^T \nabla^2 G_{i_0}(z^k) d^k \Big)
\end{aligned}
$$

are bounded. On the other hand,

$$
\gamma_{i_0}^k \Big((d^k)^T \nabla G_{i_0}(z^k) \nabla H_{i_0}(z^k)^T d^k + (d^k)^T \nabla H_{i_0}(z^k) \nabla G_{i_0}(z^k)^T d^k \Big) = -2\gamma_{i_0}^k \to -\infty
$$

by the definition of $\{d^k\}$ and (68), and

$$
\gamma_{i_0}^k \Big(G_{i_0}(z^k)(d^k)^T \nabla^2 H_{i_0}(z^k) d^k + H_{i_0}(z^k)(d^k)^T \nabla^2 G_{i_0}(z^k) d^k \Big)
$$

is bounded by the convergence of the sequences

$$
\Big\{ \gamma_{i_0}^k G_{i_0}(z^k) \Big\}, \quad \Big\{ \gamma_{i_0}^k H_{i_0}(z^k) \Big\}.
$$

In consequence, we have

$$
(d^k)^T \nabla_z^2 L_{\epsilon_k}(z^k, \lambda^k, \mu^k, \delta^k, \gamma^k) d^k \to -\infty
$$

as $k \to \infty$. This contradicts (69) and hence $\bar{z}$ is B-stationary to problem (1). □

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References

[1] G.-H. Lin and M. Fukushima, *A Modified Relaxation Scheme for Mathematical Programs with Complementarity Constraints*, Annals of Operations Research, 133 (2005), 63-84.