Performance Analysis of a Differentiated Service Router

Guidance

Professor	Masao FUKUSHIMA
Associate Professor	Tetsuya TAKINE

Masateru OHNISHI

1999 Graduate Course

in

Department of Applied Mathematics and Physics

Graduate School of Informatics

Kyoto University

February 2001

abstract

The current Internet provides only the best-effort service, which is unable to offer quality of service (QoS) guarantees. Recently, in order to offer QoS guarantees for multimedia applications such as real-time applications, Integrated Service (IntServ) was proposed. The IntServ is based on reservation to provide the end-to-end QoS guarantees per flow. The mechanism is implemented by a signaling protocol known as RSVP. The IntServ has a number of attractive features. However, its implementation has several drawbacks such as a scalability problem. More recently, Differentiated Service (DiffServ) was proposed as a scalable low-cost architecture to solve the problems in the IntServ. In the DiffServ, all flows are divided into several classes and flows in each class are aggregated. The DiffServ then provides QoS guarantees for the aggregated flows. However, it is not clear about performance the DiffServ provide for each class. Thus, quantifying such performance is very important to understand the benefits and drawbacks of the scheme. Our goal in this thesis is to examine QoS performance quantitatively.

For this purpose, we consider a Differentiated Service router model with four classes in terms of a nonpreemptive priority M/G/1 queue with repeated vacations. Packets of the first three classes are accommodated in their dedicated waiting room with finite capacity, and vacations represent services of packets with the lowest priority. The router is operated under the RIO (RED with in and out packets). For this model, The marginal queue length distribution is analyzed by constructing an embedded Markov chain. We obtain expressions for performance measures that characterize the different levels of the service such as throughput and average delay. Further, we propose a way to determine RIO parameters. In numerical experiments, we demonstrate that our proposed scheme works well to offer strict QoS guarantees for respective classes.

Contents

1 Introduction

1.1 Service of the Internet

The Internet was originally developed as a technology to link computers on campus, and rapidly expanded in the nineties. The Internet has already been one of the main communication infrastructures. There are some characteristics of the Internet as follows: First, the Internet keeps robustness against partial breakdowns in networks. Second, the Internet employs packet telecommunications, which are efficient for data and different from telephone networks. It is known that communications of the Internet are named as connection-less because edge routers put a destination address on packets and transmit them without establishing connections. Today, it is very easy to connect computers with networks owing to recent advances in end-to-end transmission control architectures (e.g., address management, routing control and mapping to IP packets) and widespread broadcasting technologies. Recently, transmission technologies as optical fiber communications have made great progress. Accordingly, the capacity of the Internet, such as bandwidth of the backbone, will be remarkably expanded in the near future. The Internet will have to provide diverse kinds of services and be shared by many users with the request of various services. However, there are still many remaining problems for the Internet to be a core network technology in the next generation.

1.2 QoS guarantees in the today's Internet

1.2.1 Current situations and problems in the Internet

As explained above, the Internet has provided only the best-effort service, which is unable to offer QoS guarantees and transmits packets without reservations of bandwidth. This service makes router's cost low because of simple transactions of packets. From the reason, this service is the majority of service in the Internet. The connection using the best-effort service efficiently utilizes the network bandwidth when the network dose not become congested. But once the network becomes congested, packets is often dropped on the account that the transmission rate is made small by the flow control. Consequently, congestion leads to the severe deterioration of performance. Thus, it is difficult to support the real-time applications (e.g., interactive voice, TV conference), which require a constant communication bandwidth during communication. Moreover, packets that arrive at a relay router are rejected when the buffer is full. Since the packet loss increases at congestion, this service cannot provide performance guarantees on the loss rate. Also it is pointed out that those users who behave in an overly aggressive way have a serious impact on other users.

1.2.2 Demands in the future Internet

As mentioned above, there are problems to provide real-time applications with end-to-end QoS guarantees over the Internet. Today, services in the Internet need to support both conventional applications (e.g., e-mail, Web) and new applications (e.g., Electronic Commerce, interactive voice and video communications). Since the problems are not new, it is unsurprising that there have been several attempts to improve them. For example, priority control schemes have already been proposed as node architecture. This preferentially transacts real-time traffic in comparison with other traffic. But when high priority traffic increases in a router, it is consequently a fatal influence on low priority traffic because of the deterioration of performance. Therefore, it is difficult to offer strict QoS guarantees by the control.

A new architecture is necessary to provide the end-to-end QoS guarantees for real-time communication services. To realize the requirements from real-time applications, there has been a major effort within the Internet Engineering Task Force (IETF) under the name of Integrated-Services (IntServ) [1, 2].

The purpose of the IntServ is to integrate non-real-time and real-time communication services over the Internet. Every flow is provided with reserved bandwidth during connection in the IntServ Packet Network. The mechanism is implemented by a resource reservation protocol known as RSVP [3, 4]. In the IntServ the influence on each reservation is not given by other users.

However, several drawbacks of the IntServ are pointed out recently. The IntServ has a scalability problem. Since each flow makes reservation at intermediate routers, it seems to be difficult immediately to handle a vast number of flows in future widespread high-speed networks. Also, the production cost of the high-speed large-scale router will become enormous and it seems to be unfavorable. Thus, it is necessary to be applicable to the widespread high-speed networks.

In 1998, to solve the problems in the IntServ, the IETF has proposed a new architecture, called differentiated services (DiffServ) [6, 7, 8]. The basic principle of the DiffServ is that network traffic is divided into a few different classes of QoS without achieving per-flow QoS guarantees. Then, the DiffServ is considered to be more scalable and lightweight in comparison to the IntServ. It builds on the classification and admission control mechanisms developed for the IntServ. But, there are few additional functions to routers.

1.3 Differentiated Service

The DiffServ is a new architecture for providing scalable service differentiation over the Internet. It is not by managing per-flow QoS but by aggregating flows that this architecture achieves scalability. Note here that Internet Service Providers (ISPs) must consider how to aggregate flows. In the DiffServ domain (DS), each packet has a particular value in the header, which is known as the Differentiated Service Code Point (DSCP). This value relies on the type of behavior aggregates (BA). This value is used to select a corresponding behavior at each hop from router to router. Thus packets with the same DSCP are treated in the same way in the DS. It is known as a Per-Hop-Behavior (PHB). Now, the IETF standardizes the four PHB classifications: expedited forwarding (EF), assured forwarding (AF), class selector (CS) and default (Best-effort). PHBs may be specified in terms of their resources (e.g., buffer size, bandwidth), priority relative to other PHBs, or in terms of their relative observable traffic characteristics (e.g., delay, loss) [9]. One difficulty here is that the parameters, which define good quality, are not self-evident in the case of the employment. It is future work to make them clear. Moreover, We have to discuss precise service level agreements (SLA) to exchange traffic both between customers and ISPs, and between ISPs. There is much to be learned about how to achieve such agreements in practice.

1.3.1 QoS guarantee architectures on the DiffServ

In principal, the DiffServ is fundamentally a very simple, efficient and highly scalable model. We describe two network architectures of the DiffServ, namely the Assured Service scheme (AS) [9] and the Premium Service scheme (PS) [8]. In 1993, Clark and Fang proposed AS to assure a guaranteed or expected throughput [9], which is the one quality of service feature to most users and applications. This measure can be used to define the minimum transfer rate.

The proposed way to assure the measure is to define a service profile at the source, which shows how packets should be transferred. The tag is set to 1 if the packet is sent according to the profile decided beforehand. The tagged packet is referred to as an in-packet. Otherwise, The tag is set to 0 (out-packet). The mechanism is implemented to drop probabilistically in/out packets in the case of congestion. As a result, the desired expected throughput is assured. The dropping mechanism of the AS scheme builds on Random Early Detection (RED) [5]. The idea of RED is to drop packets with a probability that depends on the average queue length in routers. On the other hand, Nichols and Jacobson proposed Premium Service scheme [8], which is expected to provide significant delay reduction to users and applications. The idea of the PS scheme is that routers implement several levels of priority queueing to distinguish delay. Packets are transmitted at edge routers according to their priority. Moreover, other schemes have been proposed in many fields but in practice it still has difficulty in implementing them to offer some QoS guarantees in the Internet.

So far some attempts have been made analytically to characterize the queueing model of network communications. For example, an infinite-buffer queueing model have been studied by Hayman [12]. However, for many practical communications, finite-buffer queueing models seem to be more realistic, especially for applications where it is a major concern about QoS (e.g., delay and packet loss). Finite-buffer models with vacations and no priorities have been treated by Courtois [13]. Kramer obtains recursive expressions for both the time dependent and steady state queue length distribution for an M/G/1 finite capacity queueing model with priorities and no vacations, and with a service time distribution that depends on the priority class [14]. Moreover, May obtains the steady state distribution of the queue length for an M/G/1 finite capacity queue with vacations and priorities [15]. In [15], May has analytically modeled the system, which has been proposed to solve the issues of the IntServ. The AS and the PS are combined with two kinds of PHBs in the model.

In this thesis, we consider a DiffServ router to implement the DiffServ discussed above. We assume that this router defines the four kinds of PHBs, so that this router has four finite buffers (Fig. 1). Moreover, based on the idea of the AS and the PS, we model this router a by using a finite capacity M/G/1 with vacations and priorities. As a result, this router can treat differently QoS parameters of throughput and delay. Packets are grouped into classes corresponding to their priorities. Packets of each class arrive at the system according to a Poisson process. Packets are served according to their priorities according in a non-preemptive way. Service times are independent and identically distributed according to a general distribution function, and assumed to be constant in numerical experiments.

We first analyze a first passage time and a busy period of each class in this system. Next we build up an embedded Markov chain of particular events, where packets of interest arrive and depart. Then, we analyze it with avoidance of the complexity. Based on the result, we derive the queue length distribution of a particular class at three kinds of epochs (i.e., the beginning of a service, the end of a service and the steady state). We derive QoS measures (e.g., throughput, mean queue length, mean waiting time and loss probability). Further, we consider a simple scenario of the DiffServ as mentioned in Section 5. Under the scenario, we define the specific traffic profile, which is used to provide discrimination on throughput and mean waiting time. We conclude this DiffServ performance with verifying the validity of calculation.

The rest of this thesis is organized as follows. In Section 2, we describe a DiffServ router model and the packet's arrivals. Moreover, we derive by means of the unifomizition technique a probability that some packets arrive at the system during a time interval. In Section 3, we consider a first passage time and a busy period of class 1, 2. In Section 4, based on the result, we obtain the marginal queue length distributions. In Section 5, we discuss a scenario to distinguish QoS guarantees with the router according to the priority. Then, we propose the design to provide performance discrimination of three classes as defined in our scenario and provide the results of the numerical calculation. Finally, Section 6 concludes the thesis.

2 Model description

In this section, we first describe a Differentiated Service router model in this thesis. As explained in Section 1.3, four PHB classifications are standardized, known as EF, AF, CS and Default. We consider a router with four separable queues. We assume in our router model that EF, AF and CF respectively correspond to Class 1 to 3, and Default corresponds to vacations. The assumption of vacations implies that Default class is always full.

2.1 Differentiated Service router model

We consider three finite queues, each having capacity N_i ($i = 1, \cdots, 3$). We now define ith finite queue as class i. Class i buffer is accessible only to class i packets. Refer to Figure 1. Let $G(t)$ be the probability distribution function of service times (regardless the class). Class i packets have priority over packets with larger class indices. Packets are served in a non-preemptive way, i.e., the server selects a packet with the highest priority after service completion. Within a class, packets are served on a FCFS basis. When there are no packets, the server takes repeated vacations. The durations of vacation have the same probability distribution function $G(t)$ as service times.

From a point of view of class k packets, the various classes may be grouped into three composite classes denoted by class a, (i.e., all packets with a priority higher than the priority of class k), class b, (i.e., all packets with a priority lower than the priority of class k and vacations) and finally class k itself. Note that the packets of class a are not equivalent since the rate at which they are allowed to the system depends on the class they belong to and on the state of the other queues.

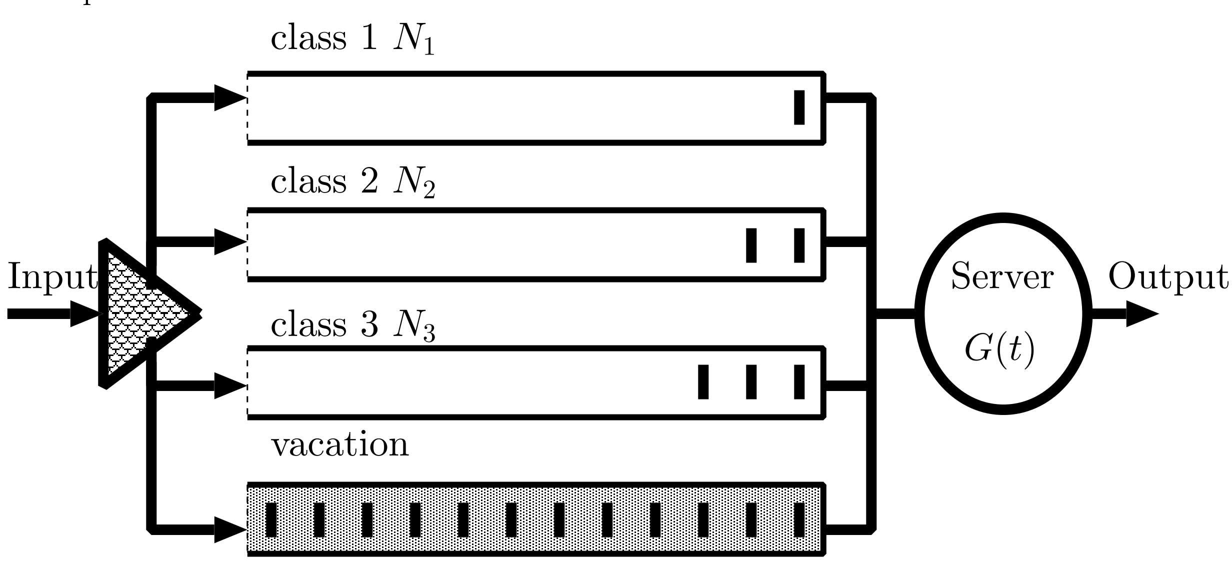

Figure 1: Differentiated Service router model.

2.2 Arrivals in Differentiated Service router model

In this subsection, we consider the arrival process treated in this thesis. As mentioned previously, it is necessary to be defined to each class a traffic profile at which each performance (e.g., bandwidth) is described in detail to provide several levels of QoS guarantees. But in practical communications of the Internet, there is a great possibility that the amount of packets exceeds the amount decided by the profile beforehand. In this thesis, if a packet is sent according to a certain profile, then we consider the packet as an *In*-packet. Otherwise, we consider it as *Out*-packet. we consider the following arrival model for the two kinds of packets, i.e., *In*-packets and *Out*-packets.

Assumption 1 *Let the stream of two kinds of arrivals to class* i $(i = 1, \cdots, 3)$*, be respectively a Poisson process with arrival rate* $\lambda_{In}^{(i)}$ *and* $\lambda_{Out}^{(i)}$ $(i = 1, \cdots, 3)$.

In what follows, we propose the active buffer management in the model. *In*-packets and *Out*-packets are dropped with a probability function depending on each queue length n_i, respectively, i.e., $p_{In}^{(i)}(n_i)$ and $p_{Out}^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)}, n_i)$ $(i = 1, \cdots, 3)$. Given that n_i packets are already in class i,

$$\lambda_{In}^{(i)}\Big(1 - p_{In}^{(i)}(n_i)\Big) + \lambda_{Out}^{(i)}\Big(1 - p_{Out}^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)}, n_i)\Big) \tag{1}$$

are accepted. From the assumption 1 and the active buffer management, we can analytically regard the two streams as one modified stream by superposition of independent *In*-packets process and *Out*-packets process. Thus, there exists three arrival streams corresponding to class. We assume the arrival rate from the stream of class i $(i = 1, \cdots, 3)$ as follows.

Assumption 2 *When there are* n_i *packets in class* i*, the arrival stream of class* i *is governed by the modified arrival process with arrival rate* $\lambda_{n_i}^{(i)}$ $(i = 1, \cdots, 3)$ *equal to* (1).

Moreover, we make the following assumption on the arrival rate.

Assumption 3 $\lambda_0^{(i)} \geq \cdots \geq \lambda_{N_i}^{(i)}, \quad i = 1, \cdots, 3.$

The assumption 3 is based on the idea of RED to avoid congestion to some extent. However, it is not analytically very significant to make this assumption except the stability of the system.

We first consider birth process for th class α $(\alpha = 1, \cdots, 3)$ arrival process.

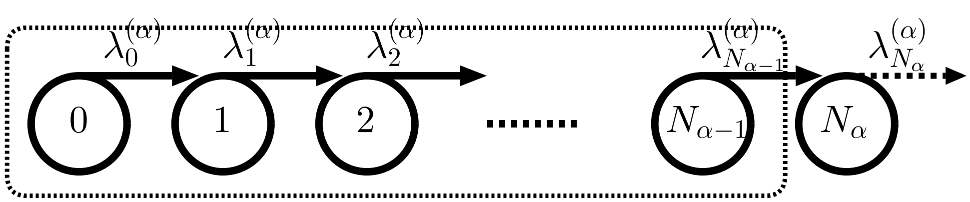

Figure 2: Arrival process.

Let $q_{i,j}^{(\alpha)}(t)$ be the conditional probability that in class α $(\alpha = 1, \cdots, 3)$ queue, a transition from i $(i = 0, \cdots, N_\alpha)$ packets to j $(j = i, \cdots, N_\alpha)$ packets happens during time interval t.

Let $\boldsymbol{Q}^{(\alpha)}$ be an $(N_\alpha+1)\times(N_\alpha+1)$ matrix whose (i,j)th element $Q^{(\alpha)}_{i,j}$ is given by

$$Q^{(\alpha)}_{i,j} = \begin{cases} -\lambda^\alpha_i, & i=j=0,\ldots,N_\alpha-1, \\ \lambda^\alpha_i, & i=0,1,\ldots,N_\alpha-1,\ j=i+1, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Moreover, let $C^{(\alpha)}(t)$ denote the number of class α packets at time t. Using (2), we have

$$q^{(\alpha)}_{i,j}(t) = \Pr\left(C^{(\alpha)}(t)=j \mid C^{(\alpha)}(0)=i\right) = [\exp(\boldsymbol{Q}^{(\alpha)}t)]_{i,j}, \qquad \begin{matrix} i=0,1,\ldots,N_\alpha-1, \\ j=i,i+1,\ldots,N_\alpha-1. \end{matrix} \tag{3}$$

To obtain $q^{(\alpha)}_{i,j}(t)$, we consider $[\exp(\boldsymbol{Q}^{(\alpha)}t)]$. Then we have by the uniformization technique

$$\begin{aligned} [\exp(\boldsymbol{Q}^{(\alpha)}t)] &= e^{-\theta^{(\alpha)}t}\exp[(\theta^{(\alpha)}\boldsymbol{I}+\boldsymbol{Q}^{(\alpha)})t] \\ &= \sum_{k=0}^{\infty} e^{-\theta^{(\alpha)}}\frac{(\theta^{(\alpha)}t)^k}{k!}[\boldsymbol{I}+\frac{1}{\theta^{(\alpha)}}\boldsymbol{Q}^{(\alpha)}]^k \\ &= \sum_{k=0}^{\infty}\gamma^{(\alpha)}_k(t)[\boldsymbol{I}+\frac{1}{\theta^{(\alpha)}}\boldsymbol{Q}^{(\alpha)}]^k, \end{aligned} \tag{4}$$

where

$$\theta^{(\alpha)} \equiv \max\{\boldsymbol{Q}^{(\alpha)}_{i,i}, i=0,1,\ldots,N_\alpha\} \quad \text{and} \quad \gamma^{(\alpha)}_k(t) \equiv e^{-\theta^{(\alpha)}}\frac{(\theta^{(\alpha)}t)^k}{k!}.$$

However, it is difficult to calculate (4) because of including infinite sum. Therefore, we approximately calculate (4) to the sufficient large K, where $\sum_{k=0}^{K}\gamma^{(\alpha)}_k(t) \simeq 1$. Thus, we have (3).

Let $\mathrm{Ar}^{(\alpha)}_{i_\alpha|n_\alpha}(t)$ $(n_\alpha = 0,\ldots,N_\alpha)$ denote the probability that there are i_α $(i_\alpha = 0,1,\ldots)$ arrivals of class α during time interval t under the condition that there are n_α in class α queue at time 0.

Then, we have the following relation $q^{(\alpha)}_{n_\alpha,n_\alpha+i_\alpha}(t)$ and $\mathrm{Ar}^{(\alpha)}_{i_\alpha|n_\alpha}(t)$:

$$q^{(\alpha)}_{n_\alpha,n_\alpha+i_\alpha}(t) = \begin{cases} \mathrm{Ar}^{(\alpha)}_{i_\alpha|n_\alpha}(t), & i_\alpha < N_\alpha - n_\alpha, \\ \displaystyle\sum_{i_\alpha=N_\alpha-n_\alpha}^{\infty}\mathrm{Ar}^{(\alpha)}_{i_\alpha|n_\alpha}(t), & i_\alpha = N_\alpha - n_\alpha. \end{cases}$$

For simplicity in notation, we define $A^{(\alpha)}_{i_\alpha|n_\alpha}(t)$ as follows:

$$A^{(\alpha)}_{i_\alpha|n_\alpha}(t) \equiv \begin{cases} q^{(\alpha)}_{n_\alpha,n_\alpha+i_\alpha}(t), & i_\alpha = 0,\ldots,N_\alpha-n_\alpha, \\ q^{(\alpha)}_{n_\alpha,n_\alpha+i_\alpha}(t), & i_\alpha = N_\alpha-n_\alpha. \end{cases} \tag{5}$$

Note that $A^{(\alpha)}_{N_\alpha-n_\alpha|n_\alpha}(t)$ includes the information that more than $N_\alpha - n_\alpha$ arrivals, i.e., when class α queue is full.

2.3 Matrix representation

In this subsection, we consider a matrix representation for the probability of all class arrivals by using $A^{(\alpha)}_{i_\alpha|n_\alpha}(t)$, which is given by (5).

First, we define $\boldsymbol{n}_j$ and $\boldsymbol{i}_j$ ($j = 1, \ldots, \mathrm{k}$) as a nonnegative $1{\times}j$ vector, which satisfies

$$\begin{array}{lll} \boldsymbol{n}_j = (n_1, \ldots, n_j), & n_i = 0, 1, \ldots, N_i, & i = 1, \ldots, j, \\ \boldsymbol{i}_j = (i_1, \ldots, i_j), & i_i = 0, 1, \ldots, N_i - n_i, & i = 1, \ldots, j. \end{array}$$

Let $A_{\boldsymbol{i}_k|\boldsymbol{n}_k}(t)$ denote the matrix probability function that a service time is less than t and there are $\boldsymbol{i}_\mathrm{k}$ arrivals during the service time under the condition that there are $\boldsymbol{n}_\mathrm{k}$ packets.

In this thesis, we use the following notation to denote the Laplace-Stieltjes transform (LST) of a probability distribution function, e.g.,

$$H^*(s) = E[e^{-st}] = \int_0^\infty dH(t), \quad Re(s) > 0.$$

We define LST of $A_{\boldsymbol{i}_k|\boldsymbol{n}_k}(t)$ as $A^*_{\boldsymbol{i}_k|\boldsymbol{n}_k}(s)$. We then have the following lemma:

Lemma 1

$$A_{\boldsymbol{i}_k|\boldsymbol{n}_k}(t) = \int_0^t \prod_{j=1}^k A^{(j)}_{i_j|n_j}(y)dG(y), \tag{6}$$

$$A^*_{\boldsymbol{i}_k|\boldsymbol{n}_k}(s) = \int_0^\infty e^{-st} dA_{\boldsymbol{i}_k|\boldsymbol{n}_k}(t). \tag{7}$$

We consider the case that service time S_v is constant, i.e.,

$$G(y) = \begin{cases} 0 & y < S_v, \\ 1 & y \geq S_v. \end{cases} \tag{8}$$

We then have the following corollary from (6), (7) and (8):

Corollary 1

$$A^*_{\boldsymbol{i}_k|\boldsymbol{n}_k}(s) = e^{-sS_v} \prod_{j=1}^k A^{(j)}_{i_j|n_j}(S_v).$$

Next, we consider a matrix representation for the stochastic matrix including the information of a transition of packets. Under the condition that the state of packets from class 1 to class j ($j = 2, \ldots, k-1$), i.e., $\boldsymbol{n}_j$ and $\boldsymbol{i}_j$ is given, we define $\boldsymbol{A}^{(j)}_{\boldsymbol{i}_j|\boldsymbol{n}_j}(s)$ as a blocking $(N_{j+1}+1)(N_{j+2}+1)\cdots(N_k+1) \times (N_{j+1}+1)(N_{j+2}+1)\cdots(N_k+1)$ matrix. We then give the following recursive formulas,

$$\boldsymbol{A}^{(k-1)}_{\boldsymbol{i}_{k-1}|\boldsymbol{n}_{k-1}}(s) = \begin{pmatrix} A^*_{(\boldsymbol{i}_{k-1},0|\boldsymbol{n}_{k-1},0)}(s) & A^*_{(\boldsymbol{i}_{k-1},1|\boldsymbol{n}_{k-1},0)}(s) & \cdots & A^*_{(\boldsymbol{i}_{k-1},N_k-1|\boldsymbol{n}_{k-1},0)}(s) & A^*_{(\boldsymbol{i}_{k-1},N_k|\boldsymbol{n}_{k-1},0)}(s) \\ 0 & A^*_{(\boldsymbol{i}_{k-1},0|\boldsymbol{n}_{k-1},1)}(s) & \cdots & A^*_{(\boldsymbol{i}_{k-1},N_k-2|\boldsymbol{n}_{k-1},1)}(s) & A^*_{(\boldsymbol{i}_{k-1},N_k-1|\boldsymbol{n}_{k-1},1)}(s) \\ 0 & 0 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & A^*_{(\boldsymbol{i}_{k-1},0|\boldsymbol{n}_{k-1},N_3-1)}(s) & A^*_{(\boldsymbol{i}_{k-1},1|\boldsymbol{n}_{k-1},N_3-1)}(s) \\ 0 & 0 & \cdots & 0 & A^*_{(\boldsymbol{i}_{k-1},0|\boldsymbol{n}_{k-1},N_3)}(s) \end{pmatrix},$$

$$
\boldsymbol{A}^{(j)}_{\boldsymbol{i}_j|\boldsymbol{n}_j}(s) = \begin{pmatrix}
\boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,0|\boldsymbol{n}_j,0)}(s) & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,1|\boldsymbol{n}_j,0)}(s) & \cdots & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,N_{j+1}-1|\boldsymbol{n}_j,0)}(s) & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,N_{j+1}|\boldsymbol{n}_j,0)}(s) \\
0 & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,0|\boldsymbol{n}_j,1)}(s) & \cdots & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,N_{j+1}-2|\boldsymbol{n}_j,1)}(s) & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,N_{j+1}-1|\boldsymbol{n}_j,1)}(s) \\
0 & 0 & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,0|\boldsymbol{n}_j,N_3-1)}(s) & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,1|\boldsymbol{n}_j,N_{j+1}-1)}(s) \\
0 & 0 & \cdots & 0 & \boldsymbol{A}^{(j+1)}_{(\boldsymbol{i}_j,0|\boldsymbol{n}_j,N_{j+1})}(s),
\end{pmatrix},
$$

for all $j = 1, 2, \ldots, k-2$.

3 Analysis of class 1 and 2 busy periods

In this section, we consider a class a busy period from a point of view of class 3, i.e., k = 3. Note that class a corresponds to class 1 and class 2, and class b to vacations. First, let us explain how a class 1 and 2 busy period is initiated. Consider a time interval during which a class 3 packet is being served or a vacation is being taken, i.e., [(a),(c)) in Figure 3. Suppose that during this time interval (b), there is at least one class 1 or 2 arrival. So, at the end of the class 3 service or the vacation, i.e., at time epoch (c), there are some backlogs of packets of class 1 and 2. As soon as the class 3 service or the vacation is finished, one of class 1 or 2 packets in the system according to priority, is being selected in a FCFS way for service. The start of the service of this packet corresponds to the beginning of a class 1 and 2 busy period. This period terminates when both class 1 queue and class 2 queue are empty, i.e., at time epoch(d). But if there is no class 1 and 2 arrival during the initial class 3 packet is being served or the vacation, then a class 1 and 2 busy period does not follow the service.

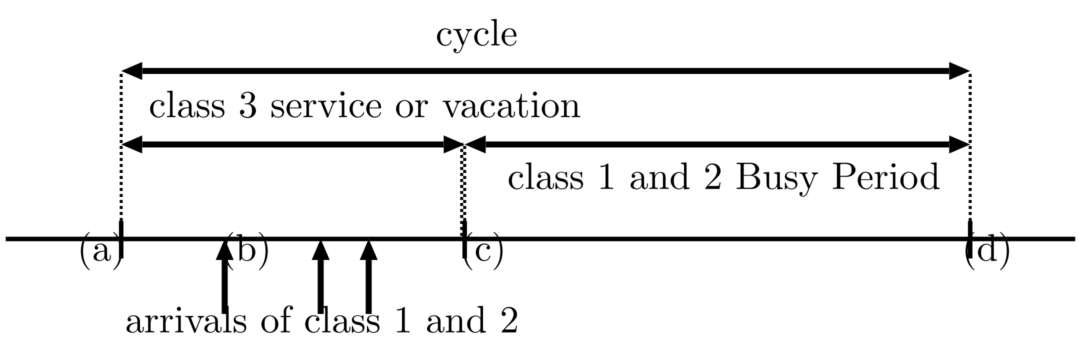

Figure 3: Class 1 and 2 busy periods.

In what follows, we explain each class first passage time to obtain a class 1 and 2 busy period. We define $\boldsymbol{B}^{(1,i_1)}(t)$ ($i_1 = N_1, \ldots, 1$) as an $(N_2+1)(N_3+1) \times (N_2+1)(N_3+1)$ stochastic matrix, which means the probability distribution function of the first passage time of class 1 to state (i_1-1, i_2', i_3') during less than t, given that it starts from state (i_1, i_2, i_3) for $i_2 = 0, \ldots, N_2$, $i_3 = 0, \ldots, N_3$, $i_2' = i_2, \ldots, N_2$ and $i_3' = 0, \ldots, N_3$. Also, we define $\boldsymbol{B}^{(2,i_2)}(t)$ ($i_2 = N_2, \ldots, 1$) as an $(N_3+1) \times (N_3+1)$ stochastic matrix, which means the probability distribution function of the first passage time of class 2 to state $(0, i_2 - 1, i_3^{'})$ during less than t, given that it starts from state $(0, i_2, i_3)$ for $i_3 = 0, \ldots, N_3$ and $i_3^{'} = 0, \ldots, N_3$.

Let $\boldsymbol{B}^{(1,i_1)}(s)$ (resp. $\boldsymbol{B}^{(2,i_2)}(s)$) be an $(N_2+1)(N_3+1) \times (N_2+1)(N_3+1)$ (resp. $(N_3+1) \times (N_3+1)$) matrix whose element is LST of $\boldsymbol{B}^{(1,i_1)}(t)$ (resp. $\boldsymbol{B}^{(2,i_2)}(t)$), where

$$\boldsymbol{B}^{(1,i_1)}(s) = \int_0^\infty e^{-st} d\boldsymbol{B}^{(1,i_1)}(t), \quad \boldsymbol{B}^{(2,i_2)}(s) = \int_0^\infty e^{-st} d\boldsymbol{B}^{(2,i_2)}(t).$$

3.1 First passage time of class 1

If there are some class 1 packets at the end of a service, then a class 1 packet is preferentially served. Therefore, we only consider class 1 services. We then have the following lemma for class 1 first passage time;

Lemma 2

$$\boldsymbol{B}^{(1,i)}(s) = \begin{cases} \boldsymbol{A}^{(1)}_{0|N_1}(s), & i = N_1, \\ \boldsymbol{A}^{(1)}_{0|i}(s) + \sum_{i_1=1}^{N_1-i} \boldsymbol{A}^{(1)}_{i_1|i}(s)\boldsymbol{B}^{(1,i+i_1-1)}(s) \cdots \boldsymbol{B}^{(1,i)}(s), & i = 1, \cdots, N_1 - 1. \end{cases} \tag{9}$$

Proof. To obtain (9), we need to consider the following two situations.

Case (1): There are N_1 packets at the start of a class 1 packet's service, i.e., full.

Case (2): There are less than N_1 packets at the start of a class 1 packet's service, i.e., not full.

In Case (1), class 1 packets who arrive during a class 1 service are only dropped. So when the service is finished, there are $N_1 - 1$ packets in class 1 queue. As a result, we have

$$\boldsymbol{B}^{(1,N_1)}(s) = \boldsymbol{A}^{(1)}_{0|N_1}(s). \tag{10}$$

In Case (2), when there are no class 1 arrivals during a class 1 service, we have immediately $\boldsymbol{A}^{(1)}_{0|i}(s)$. Next, when there are some class 1 arrivals during a class 1 service, class 1 first passage time is lengthened to serve class 1 new packets. As a result, we have

$$\sum_{i_1=1}^{N_1-i} \boldsymbol{A}^{(1)}_{i_1|i}(s)\boldsymbol{B}^{(1,i+i_1-1)}(s) \cdots \boldsymbol{B}^{(1,i)}(s). \tag{11}$$

Therefore, we obtain (9) from (11) and (10). □

Moreover, we define $\boldsymbol{B}^{(1,0)}(s)$ as an $(N_2+1)(N_3+1) \times (N_2+1)(N_3+1)$ identity matrix for simplicity.

3.2 First passage time of class 2

In this subsection, we consider class 2 first passage time. When there is no a class 1 packet at the end of a service, a class 2 packet gets served. However, if some class 1 packets arrive during the service, then the next class 2 packet, if any, has to wait until the end of class 1 busy period (recall the service discipline). Therefore, we obtain the following lemma for class 2 first passage time:

Lemma 3

$$\boldsymbol{B}^{(2,i)}(s) = \sum_{i_1=0}^{N_1}\Big(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i-1\ \text{elements}}, \boldsymbol{A}^{(2)}_{(i_1,0|0,i)}(s), \boldsymbol{A}^{(2)}_{(i_1,1|0,i)}(s), \cdots,$$

$$\boldsymbol{A}^{(2)}_{(i_1,N_2-i-1|0,i)}(s), \boldsymbol{A}^{(2)}_{(i_1,N_2-i|0,i)}(s), \boldsymbol{O}\Big)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s), \qquad (12)$$

$$i = 1,\cdots,N_2,$$

where $\boldsymbol{O}$ denotes an appropriate zero matrix, $\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)$ are given by

$$\widetilde{\boldsymbol{B}}^{(1,i)}(s) = \begin{cases} \boldsymbol{I}, \quad \text{identity matrix}, & i = 0, \\ \boldsymbol{B}^{(1,i-1)}(s)\cdots\boldsymbol{B}^{(1,1)}(s)\boldsymbol{B}^{(1,0)}(s), & i = 1,\ldots,N_1, \end{cases}$$

and $\boldsymbol{H}_i(s)$ is defined as the following $(N_2+1)(N_3+1)\times(N_2+1)$ nonnegative matrix:

$$\boldsymbol{H}_i(s) = \begin{cases} \Big(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i-1\ \text{elements}}, \boldsymbol{I}, \boldsymbol{B}^{(2,i)}(s), \boldsymbol{B}^{(2,i+1)}(s)\boldsymbol{B}^{(2,i)}(s), \cdots, \boldsymbol{B}^{(2,N_2)}(s)\cdots\boldsymbol{B}^{(2,i)}(s)\Big)^{\mathbf{T}}, & \\ & i = 1,\cdots,N_2, \\ \Big(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{N_2\ \text{elements}}, \boldsymbol{I}\Big)^{\mathbf{T}}, & i = N_2+1. \end{cases}$$

Proof. To obtain (12), we have to consider only the following three situations.
Case (1): There are no class 1 and class 2 arrivals during an initial class 2 service.
Case (2): There are no class 1 arrivals but some class 2 arrivals during an initial class 2 service.
Case (3): There are some class 1 arrivals during an initial class 2 service.
In Case (1), we immediately obtain

$$\boldsymbol{A}^{(2)}_{(0,0|0,i)}(s), \quad i = 1,\ldots,N_2. \qquad (13)$$

In Case (2), we consider only class 2 arrivals under the assumption of no class 1 arrivals during an initial class 2 service. So, we have the following terms in the similar way to class 1 first passage time:

$$\sum_{i_2=1}^{N_2-i}\boldsymbol{A}^{(2)}_{(0,i_2|0,i)}(s)\boldsymbol{B}^{(2,i+i_2-1)}(s)\cdots\boldsymbol{B}^{(2,i)}(s), \quad i = 1,\ldots,N_2. \qquad (14)$$

Finally, in Case (3), we have the following terms:

$$\sum_{i_1=1}^{N_1}\sum_{i_2=0}^{N_2-i}\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i+i_2-1\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,i_2|0,i)}(s),\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{N_2-i-i_2+1\ \text{elements}}\Bigg)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\begin{pmatrix}\left.\begin{matrix}\boldsymbol{O}\\ \vdots\\ \boldsymbol{O}\end{matrix}\right\}i-1\ \text{elements}\\ \boldsymbol{I}\\ \boldsymbol{B}^{(2,i)}(s)\\ \boldsymbol{B}^{(2,i+1)}(s)\boldsymbol{B}^{(2,i)}(s)\\ \vdots\\ \boldsymbol{B}^{(2,N_2)}(s)\cdots\boldsymbol{B}^{(2,i)}(s)\end{pmatrix}$$

$$=\sum_{i_1=1}^{N_1}\sum_{i_2=0}^{N_2-i}\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i+i_2-1\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,i_2|0,i)}(s),\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{N_2-i-i_2+1\ \text{elements}}\Bigg)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s),\qquad i=1,\ldots,N_2. \tag{15}$$

From (13), (14) and (15), $\boldsymbol{B}^{(2,i)}(s)$ $(i=1,\cdots,N_2)$ is obtained to be

$$\begin{aligned}\boldsymbol{B}^{(2,i)}(s)&=\boldsymbol{A}^{(2)}_{(0,0|0,i)}(s)+\sum_{i_2=1}^{N_2-i}\boldsymbol{A}^{(2)}_{(0,i_2|0,i)}(s)\boldsymbol{B}^{(2,i+i_2-1)}(s)\cdots\boldsymbol{B}^{(2,i)}(s)\\&\qquad+\sum_{i_1=1}^{N_1}\sum_{i_2=0}^{N_2-i}\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i+i_2-1\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,i_2|0,i)}(s),\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{N_2-i-i_2+1\ \text{elements}}\Bigg)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s)\\&=\sum_{i_2=0}^{N_2-i}\boldsymbol{A}^{(2)}_{(0,i_2|0,i)}(s)\boldsymbol{B}^{(2,i+i_2-1)}(s)\cdots\boldsymbol{B}^{(2,i)}(s)\\&\qquad+\sum_{i_1=1}^{N_1}\sum_{i_2=0}^{N_2-i}\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i+i_2-1\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,i_2|0,i)}(s),\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{N_2-i-i_2+1\ \text{elements}}\Bigg)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s)\\&=\sum_{i_1=0}^{N_1}\sum_{i_2=0}^{N_2-i}\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i+i_2-1\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,i_2|0,i)}(s),\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{N_2-i-i_2+1\ \text{elements}}\Bigg)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s)\\&=\sum_{i_1=0}^{N_1}\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i-1\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,0|0,i)}(s),\boldsymbol{A}^{(2)}_{(i_1,1|0,i)}(s),\cdots,\\&\qquad\boldsymbol{A}^{(2)}_{(i_1,N_2-i-1|0,i)}(s),\boldsymbol{A}^{(2)}_{(i_1,N_2-i|0,i)}(s),\boldsymbol{O}\Bigg)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s),\quad i=1,\cdots,N_2.\end{aligned}$$

□

For abbreviation of notation, let $\boldsymbol{M}^i_{i_1}(s)$ be an $(N_3+1)\times(N_2+1)(N_3+1)$ matrix which is given by

$$\boldsymbol{M}^i_{i_1}(s)\equiv\Bigg(\underbrace{\boldsymbol{O},\cdots,\boldsymbol{O}}_{i-1\ \text{elements}},\underbrace{\boldsymbol{A}^{(2)}_{(i_1,0|0,i)}(s),\boldsymbol{A}^{(2)}_{(i_1,1|0,i)}(s),\cdots,\boldsymbol{A}^{(2)}_{(i_1,N_2-i-1|0,i)}(s)}_{N_2-i\ \text{elements}},\boldsymbol{A}^{(2)}_{(i_1,N_2-i|0,i)}(s),\boldsymbol{O}\Bigg),\qquad i=1,\ldots,N_2,\quad i_1=0,1,\ldots,N_1. \tag{16}$$

Then, we have $\boldsymbol{B}^{(2,i)}(s)$ in terms of $\boldsymbol{M}_{i_1}^{i}(s)$

$$\boldsymbol{B}^{(2,i)}(s) = \sum_{i_1=0}^{N_1} \boldsymbol{M}_{i_1}^{i}(s)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_i(s), \quad i = 1, \cdots, N_2.$$

Moreover, we define $\sum_{i_1=0}^{N_1} \boldsymbol{M}_{i_1}^{i}(s)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)$ as an $(N_3+1) \times (N_2+1)(N_3+1)$ matrix which satisfies

$$\boldsymbol{\Phi}_i(s) \equiv \sum_{i_1=0}^{N_1} \boldsymbol{M}_{i_1}^{i}(s)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s), \qquad i = 1, 2, \cdots, N_2.$$

Finally, we have the following equations for the class 1 and 2 first passage time:

$$\begin{aligned}
\boldsymbol{B}^{(1,N_1)}(s) &= \boldsymbol{A}_{0|N_1}^{(1)}(s), \\
\boldsymbol{B}^{(1,i)}(s) &= \boldsymbol{A}_{0|i}^{(1)}(s) + \sum_{i_1=1}^{N_1-i} \boldsymbol{A}_{i_1|i}^{(1)}(s)\boldsymbol{B}^{(1,i+i_1-1)}(s)\cdots\boldsymbol{B}^{(1,i)}(s), \quad i = 1, \cdots, N_1 - 1, \\
\boldsymbol{B}^{(2,i)}(s) &= \boldsymbol{\Phi}_i(s)\boldsymbol{H}_i(s), \qquad i = 1, \cdots, N_2.
\end{aligned}$$

Note here that $\boldsymbol{H}_i(s)$ satisfies the following relation:

$$\boldsymbol{H}_i(s) = \boldsymbol{J}_i + \boldsymbol{H}_{i+1}(s)\boldsymbol{B}^{(2,i)}(s), \quad i = 1, \cdots, N_2, \tag{17}$$

where

$$\boldsymbol{J}_i^{\mathbf{T}} = (\underbrace{\boldsymbol{O}, \cdots, \boldsymbol{O}}_{i-1 \text{ elements}}, \boldsymbol{I}, \underbrace{\boldsymbol{O}, \cdots, \boldsymbol{O}}_{N_2+1-i \text{ elements}}), \quad i = 1, \ldots, N_2.$$

Thus, using (9), (12) and (17), we have the following equation for class 1 and class 2 first passage time:

$$\begin{aligned}
\boldsymbol{B}^{(1,N_1)}(s) &= \boldsymbol{A}_{0|N_1}^{(1)}(s) \\
\boldsymbol{B}^{(1,N_1-1)}(s) &= \left(\boldsymbol{I} - \boldsymbol{A}_{1|N_1-1}^{(1)}(s)\right)^{-1} \boldsymbol{A}_{0|N_1-1}^{(1)}(s) \\
\boldsymbol{B}^{(1,i)}(s) &= \left(\boldsymbol{I} - \left(\boldsymbol{A}_{1|i}^{(1)}(s) + \sum_{i_1=2}^{N_1-i} \boldsymbol{A}_{i_1|i}^{(1)}(s)\boldsymbol{B}^{(1,i+i_1-1)}(s)\cdots\boldsymbol{B}^{(1,i+1)}(s)\right)\right)^{-1} \boldsymbol{A}_{0|i}^{(1)}(s) \\
& \qquad i = 1, \cdots, N_1 - 2, \\
\boldsymbol{B}^{(2,i)}(s) &= \left(\boldsymbol{I} - \boldsymbol{\Phi}_i(s)\boldsymbol{H}_{i+1}(s)\right)^{-1} \boldsymbol{\Phi}_i(s)\boldsymbol{J}_i \qquad i = 1, \cdots, N_2
\end{aligned} \tag{18}$$

Note that the existence of the inverse matrixes in (18) means the stability that a class 1 and 2 busy period terminates sooner or later.

4 Queue length distribution

In this subsection, We derive three kinds of marginal queue length distribution of class 3 packets.

4.1 Queue length distribution at starts of class 3 services and vacations

We consider class 3 queue length distribution at the starts of class 3 services and vacations. We denote $\boldsymbol{\nu} = (\nu_i)$ $(i = 0, 1, \ldots, N_3)$ as a $1 \times (N_3 + 1)$ stochastic vector, which means class 3 queue length distribution at the starts of class 3 services and vacations. To obtain $\boldsymbol{\nu}$, we use a classical embedded Markov chain approach. Note that the i_3'th column of $\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_1(s)$ represents the LST of the probability distribution function of the first passage time to state $(0, 0, i_3')$ given that it starts from state (i_1, i_2, i_3), where state (i_1, i_2, i_3) denotes there are i_j packets of class j $(j = 1, 2, 3)$. Here, let $\boldsymbol{X}_{|i_1,i_2}(s)$ $(i_1 = 0, \ldots, N_1,\ i_2 = 0, \ldots, N_2)$ denote an $(N_3+1)\times(N_3+1)$ matrix whose element $(\boldsymbol{X}_{|i_1,i_2}(s))_{i_3,i_3'}$ $(i_3 = 0, \ldots, N_3,\ i_3' = i_3, \ldots, N_3)$ means LST of the probability distribution function of the first passage time to state $(0, 0, i_3')$ given that it starts from state (i_1, i_2, i_3). We then have

$$\boldsymbol{X}_{|i_1,i_2}(s) \equiv \Big(\underbrace{\boldsymbol{O}, \cdots, \boldsymbol{O}}_{i_2 \text{ elements}}, \boldsymbol{I},\ \underbrace{\boldsymbol{O}, \cdots, \boldsymbol{O}}_{N_2 - i_2 \text{ elements}}\Big)\widetilde{\boldsymbol{B}}^{(1,i_1)}(s)\boldsymbol{H}_1(s), \qquad \begin{array}{l} i_1 = 0, \ldots, N_1, \\ i_2 = 0, \ldots, N_2. \end{array} \tag{19}$$

In what follows, we consider a transition matrix at embedded Markov epochs.

Lemma 4 *Suppose* $\boldsymbol{X}_{|i_1,i_2}(s)$ $(i_1 = 0, \ldots, N_1,\ i_2 = 0, \ldots, N_2)$ *is given by (19). Let* $\boldsymbol{P}(s)$ *denote an* $(N_3 + 1) \times (N_3 + 1)$ *stochastic matrix whose* (i, j)*th element* $P_{i,j}(s)$ *is given by*

$$\begin{aligned}
P_{0,j}(s) &= \sum_{i_1=0}^{N_1}\sum_{i_2=0}^{N_2}\sum_{i_3=0}^{j} A^*_{i_1,i_2,i_3|0,0,0}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i_3,j}, && j = 0, 1, \ldots, N_3, \\
P_{i,j}(s) &= \sum_{i_1=0}^{N_1}\sum_{i_2=0}^{N_2}\sum_{i_3=0}^{j-i+1} A^*_{i_1,i_2,i_3|0,0,i}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i+i_3-1,j}, && \begin{array}{l} i = 1, \ldots, N_3, \\ j = i-1, i, \ldots, N_3 - 1, \end{array} \\
P_{i,N_3}(s) &= \sum_{i_1=0}^{N_1}\sum_{i_2=0}^{N_2}\sum_{i_3=0}^{N_3-i} A^*_{i_1,i_2,i_3|0,0,i}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i+i_3-1,N_3}, && i = 1, \ldots, N_3.
\end{aligned}$$

In what follows, we describe $\boldsymbol{P}(s)$ in a matrix form. We first note that a part of $P_{0,j}(s)$, $\sum_{i_3=0}^{j} A^*_{i_1,i_2,i_3|0,0,0}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i_3,j}$ is equal to the $(0, j)$th element of

$$\begin{aligned}
&A^{(2)}_{i_1,i_2|0,0}(s)(\boldsymbol{X}_{|i_1,i_2}(s)) = \\
&\begin{pmatrix}
A^*_{i_1,i_2,0|0,0,0}(s) & A^*_{i_1,i_2,1|0,0,0}(s) & \cdots & A^*_{i_1,i_2,N_3-1|0,0,0}(s) & A^*_{i_1,i_2,N_3|0,0,0}(s) \\
0 & A^*_{i_1,i_2,0|0,0,1}(s) & \cdots & A^*_{i_1,i_2,N_3-2|0,0,1}(s) & A^*_{i_1,i_2,N_3-1|0,0,1}(s) \\
0 & 0 & \ddots & \vdots & \vdots \\
0 & 0 & 0 & A^*_{i_1,i_2,0|0,0,N_3-1}(s) & A^*_{i_1,i_2,1|0,0,N_3-1}(s) \\
0 & 0 & 0 & 0 & A^*_{i_1,i_2,0|0,0,N_3}(s)
\end{pmatrix}\boldsymbol{X}_{|i_1,i_2}(s), \quad (20) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad j = 0, \ldots, N_3.
\end{aligned}$$

Thus, we immediately have the following equation from (20).

$$\sum_{i_2=0}^{N_2}\sum_{i_3=0}^{j} A^*_{i_1,i_2,i_3|0,0,0}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i_3,j} = \left(A^{(2)}_{i_1,0|0,0}(s), A^{(2)}_{i_1,1|0,0}(s), \cdots, A^{(2)}_{i_1,N_2|0,0}(s)\right)\begin{pmatrix} \boldsymbol{X}_{|i_1,0}(s) \\ \boldsymbol{X}_{|i_1,1}(s) \\ \vdots \\ \boldsymbol{X}_{|i_1,N_2}(s) \end{pmatrix}$$

$$= (\boldsymbol{I}, \boldsymbol{O}, \cdots, \boldsymbol{O})\, \boldsymbol{A}^{(1)}_{i_1|0}(s)\vec{\boldsymbol{X}}_{i_1}(s),$$

where

$$\vec{\boldsymbol{X}}^{\mathbf{T}}_{i_1}(s) \equiv \left(\boldsymbol{X}_{|i_1,0}(s), \boldsymbol{X}_{|i_1,1}(s), \cdots, \boldsymbol{X}_{|i_1,N_2}(s)\right) \quad i_1 = 0, \ldots, N_1. \tag{21}$$

Lemma 5 *Suppose* $\vec{\boldsymbol{X}}^{\mathbf{T}}_{i_1}(s)$ $(i_1 = 0, \ldots, N_1)$ *is given by (21).* $P_{0,j}(s)$ $(j = 0, \ldots, N_3)$ *is equal to the* $(0, j)$*th element of* $\boldsymbol{Y}(s)$*, i.e.,* $Y_{i,j}(s)$ $(i, j = 0, 1, \cdots, N_3)$*, where*

$$\boldsymbol{Y}(s) \equiv \sum_{i_1=0}^{N_1} (\boldsymbol{I}, \boldsymbol{O}, \cdots, \boldsymbol{O})\, \boldsymbol{A}^{(1)}_{i_1|0}(s)\vec{\boldsymbol{X}}_{i_1}(s). \tag{22}$$

Next, we consider $P_{i,j}(s)$ $(i = 1, 2, \ldots, N_3, j = i-1, i, \ldots, N_3 - 1)$ in a matrix form. Note here that there is one class 3 departure during an interval between the embedded Markov epochs ,i.e., a cycle. We consider $\sum_{i_3=0}^{j-i+1} A^*_{i_1,i_2,i_3|0,0,i}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i+i_3-1,j}$, which is the component of $P_{i,j}(s)$. Let $\boldsymbol{E}$ be an $(N_3 + 1) \times (N_3 + 1)$ matrix whose (i, j)th element $E_{i,j}$ is given by

$$E_{i,j} = \begin{cases} 1, & \text{if } i = 1, \ldots, N_3,\ j = i-1, \\ 0, & \text{otherwise.} \end{cases}$$

By multiplying $\boldsymbol{A}^{(2)}_{i_1,i_2|0,0}(s)$ by $\boldsymbol{E}$ from the right side, we have

$$\boldsymbol{A}^{(2)}_{i_1,i_2|0,0}(s)\boldsymbol{E} = \begin{pmatrix} A^*_{i_1,i_2,1|0,0,i}(s) & \cdots & A^*_{i_1,i_2,N_3-1|0,0,i}(s) & A^*_{i_1,i_2,N_3|0,0,i}(s) & 0 \\ A^*_{i_1,i_2,0|0,0,i}(s) & \cdots & A^*_{i_1,i_2,N_3-2|0,0,i}(s) & A^*_{i_1,i_2,N_3-1|0,0,i}(s) & 0 \\ 0 & \ddots & \vdots & \vdots & 0 \\ 0 & 0 & A^*_{i_1,i_2,0|0,0,i}(s) & A^*_{i_1,i_2,1|0,0,i}(s) & 0 \\ 0 & 0 & 0 & A^*_{i_1,i_2,0|0,0,i}(s) & 0 \end{pmatrix}.$$

We then see that the component $\sum_{i_3=0}^{j-i+1} A^*_{i_1,i_2,i_3|0,0,i}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i+i_3-1,j}$ is equal to the (i, j)th element of $\boldsymbol{A}^{(2)}_{i_1,i_2|0,0}(s)\boldsymbol{E}\boldsymbol{X}_{|i_1,i_2}(s)$. Thus, we have

$$\sum_{i_2=0}^{N_2}\sum_{i_3=0}^{j-i+1} A^*_{i_1,i_2,i_3|0,0,i}(s)(\boldsymbol{X}_{|i_1,i_2}(s))_{i+i_3-1,j}$$

$$= \left(\boldsymbol{A}^{(2)}_{i_1,0|0,0}(s)\boldsymbol{E}, \boldsymbol{A}^{(2)}_{i_1,1|0,0}(s)\boldsymbol{E}, \cdots, \boldsymbol{A}^{(2)}_{i_1,N_2|0,0}(s)\boldsymbol{E}\right)\vec{\boldsymbol{X}}_{i_1}(s)$$

$$= (\boldsymbol{I}, \boldsymbol{O}, \cdots, \boldsymbol{O})\, \boldsymbol{A}^{(1)}_{i_1|0}(s)\begin{pmatrix} \boldsymbol{E} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{E} \end{pmatrix}\vec{\boldsymbol{X}}_{i_1}(s).$$

Therefore, we see that $P_{i,j}(s)$ includes the (i,j)th element of the matrix at the right hand in (23). Here, for simplicity in notation, we define $\widetilde{\boldsymbol{A}}^{(1)}_{i_1|0}(s)$ $(i_1 = 0, \ldots, N_1)$ as an $(N_2+1)(N_3+1) \times (N_2+1)(N_3+1)$ matrix which satisfies

$$\widetilde{\boldsymbol{A}}^{(1)}_{i_1|0}(s) \equiv \boldsymbol{A}^{(1)}_{i_1|0}(s) \begin{pmatrix} \boldsymbol{E} & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{E} \end{pmatrix}. \tag{23}$$

Thus, we have the following lemma.

Lemma 6 *Suppose* $\vec{\boldsymbol{X}}^{\mathbf{T}}_{i_1}(s)$ $(i_1 = 0, \ldots, N_1)$ *is given by (21), and* $\widetilde{\boldsymbol{A}}^{(1)}_{i_1|0}(s)$ *is given by (23).* $P_{i,j}(s)$ $(i = 1, \ldots, N_3,\ j = i-1, \ldots, N_3 - 1)$ *is equal to the* (i,j)*th element of* $\boldsymbol{Z}(s)$*, i.e.,* $Z_{i,j}(s)$ $(i,j = 0, 1, \cdots, N_3)$*, where*

$$\boldsymbol{Z}(s) \equiv \sum_{i_1=0}^{N_1} (\boldsymbol{I}, \boldsymbol{O}, \cdots, \boldsymbol{O})\, \widetilde{\boldsymbol{A}}^{(1)}_{i_1|0}(s) \vec{\boldsymbol{X}}_{i_1}(s). \tag{24}$$

Finally, we consider $P_{i,N_3}(s)$ $(i = 1, \ldots, N_3)$.

We note that $\sum_{i_2=0}^{N_2} \sum_{i_3=0}^{N_3-i} A^{*}_{i_1,i_2,i_3|0,0,i}(s) (\boldsymbol{X}_{|i_1,i_2}(s))_{i+i_3-1,N_3}$ is equal to the (i,N_3)th element of $(\boldsymbol{I}, \boldsymbol{O}, \cdots, \boldsymbol{O})\, \widetilde{\boldsymbol{A}}^{(1)}_{i_1|0}(s) \vec{\boldsymbol{X}}_{i_1}(s)$. Thus, we see that $P_{i,N_3}(s)$ is equal to the (i,N_3)th element of $\boldsymbol{Z}(s)$. Consequently, we can make the following expression of $\boldsymbol{P}(s)$ from (22) and (24),

$$\boldsymbol{P}(s) = \begin{pmatrix} Y_{0,0}(s) & Y_{0,1}(s) & \cdots & Y_{0,N_3-1}(s) & Y_{0,N_3}(s) \\ Z_{1,0}(s) & Z_{1,1}(s) & \cdots & Z_{1,N_3-1}(s) & Z_{1,N_3}(s) \\ 0 & Z_{2,1}(s) & \cdots & Z_{2,N_3-1}(s) & Z_{2,N_3}(s) \\ & 0 & \ddots & \vdots & \vdots \\ \mathbf{0} & & 0 & Z_{N_3,N_3-1}(s) & Z_{N_3,N_3}(s) \end{pmatrix}.$$

Let $\boldsymbol{P}$ be the transition probability matrix whose (i,j)th element $P_{i,j}$ is given by $P_{i,j}(s)\mid_{s=0+}$. Here, we introduce the following theorem.

Theorem 1 *When* P *is irreducible, there is a unique probability vector* x*, which satisfies* $xP = x$ *and* $x\boldsymbol{e} = 1$.

By using $\boldsymbol{P}$, $\boldsymbol{\nu}$ is uniquely determined by

$$\boldsymbol{\nu} = \boldsymbol{\nu}\boldsymbol{P}, \qquad \boldsymbol{\nu}\boldsymbol{e} = 1,$$

where $\boldsymbol{e}$ denotes a $1 \times N_3 + 1$ vector whose elements are all equal to one. In what follows, we consider the class 3 queue length distribution at the departure epochs of class 3 packets.

4.2 Queue length distribution at departures of class 3 packets

In this subsection, we derive the class 3 queue length distribution $\boldsymbol{\xi}$ of class 3 packets at departures of class 3 packets. Note that there are at most $N_3 - 1$ packets immediately after departures of class 3 packets.

Recall that $A_{i_1,i_2,i_3|0,0,n_3}(t)$ denotes the probability distribution function that i_1, i_2 and i_3 packets respectively arrive during a class 3 service time t or a vacation time t, given that it starts from $(0,0,n_3)$ packets at time 0. Here, let $G_{i_3|n_3}$ be the number of class 3 packets who arrive during the service under the condition that there are present n_3 packets of class 3. Then we have

$$G_{i_3|n_3} = \sum_{i_1=0}^{N_1}\sum_{i_2=0}^{N_2}\int_0^{\infty} dA_{i_1,i_2,i_3|0,0,n_3}(y), \qquad \begin{array}{l} n_3 = 0,\ldots,N_3, \\ i_3 = 0,1,\ldots. \end{array} \tag{25}$$

Under the assumption that service time distribution function is given by (8), we have from (6)

$$G_{i_3|n_3} = A_{i_3|n_3}(S_v), \qquad \begin{array}{l} n_3 = 0,\ldots,N_3, \\ i_3 = 0,1,\ldots. \end{array}$$

We see that the queue length distribution $\boldsymbol{\xi} = \xi_j$ satisfies the following equations in terms of (25):

$$\begin{aligned} \xi_j &= \frac{\sum_{n_3=1}^{j+1} \nu_{n_3} G_{j-n_3+1|n_3}}{\sum_{i=1}^{N_3} \nu_i}, \quad j = 0,1,\ldots,N_3-2, \\ \xi_{N_3-1} &= \frac{\sum_{n_3=1}^{N_3} \nu_{n_3} \sum_{i_3=N_3-n_3}^{\infty} G_{i_3|n_3}}{\sum_{i=1}^{N_3} \nu_i}. \end{aligned} \tag{26}$$

Furthermore,by the law of total probability, we have

$$\sum_{j=0}^{N_3-1} \xi_j = \frac{\sum_{j=0}^{N_3-2}\sum_{n_3=1}^{j+1} \nu_{n_3} G_{j-n_3+1|n_3} + \sum_{n_3=1}^{N_3} \nu_i G_{\overline{N_3-n_3}|n_3}}{\sum_{i=1}^{N_3} \nu_i} = 1, \tag{27}$$

where notation $\bar{x}_j$ means $\sum_{i=j}^{\infty} x_i$. Thus, $\boldsymbol{\xi}$ is uniquely decided by (26) and (27). Finally, we consider the steady state distribution $\boldsymbol{\pi}$ of the length of class 3.

4.3 The steady state queue length distribution

By using $\boldsymbol{\xi}$ obtained above, we derive the steady state queue length distribution of class 3. When a class 3 packet arrives at the queue, the packet sees the state that from 0 to N_3 packets are waiting for a service. Here, note that the queue rejects the class 3 new packets who see the state that there are already present N_3 packets of class 3. We have the following probability that there are j packets of class 3 in the queue:

$$\frac{\lambda_j^{(3)}\pi_j}{\sum_{j=0}^{N_3}\lambda_j^{(3)}\pi_j}, \quad j = 0, \ldots, N_3.$$

Therefore, we have the loss probability of class 3 packets, i.e.,

$$\frac{\lambda_{N_3}^{(3)}\pi_{N_3}}{\sum_{j=0}^{N_3}\lambda_j^{(3)}\pi_j}. \tag{28}$$

Since the queue length distribution formed by arrivals is identical to that at departures, we have

$$\xi_j = \frac{\dfrac{\lambda_j^{(3)}\pi_j}{\sum_{j=0}^{N_3}\lambda_j^{(3)}\pi_j}}{1 - \dfrac{\lambda_{N_3}^{(3)}\pi_{N_3}}{\sum_{j=0}^{N_3}\lambda_j^{(3)}\pi_j}}, \quad j = 0, \ldots, N_3 - 1. \tag{29}$$

Next, we consider the utilization factor of class 3 packets. We assume that the random variable F representing the length of a cycle, i.e., from the start of a service of a class 3 packet or a vacation to termination of the following class 1 and 2 busy period, is distributed according to a probability distribution function $F(t)$, with LST $F^*(s)$. We then have

$$F^*(s) = E[e^{-sF}] = \int_0^\infty e^{-st}dF(t), \quad Re(s) > 0.$$

Thus, we have

$$F^*(s) = \nu_0 \sum_{j=0}^{N_3} P_{0,j}(s) + \sum_{i=1}^{N_3}\sum_{j=i-1}^{N_3} \nu_i P_{i,j}(s). \tag{30}$$

Let $\overline{G}$ and $\overline{F}$ denote the average lengths of service times and of cycles, respectively, i.e.,

$$\overline{G} = \int_0^\infty (1 - G(x))dx, \quad \overline{F} = -\frac{d}{ds}(F^*(s))\bigg|_{s=0+},$$

where the latter is obtained by differentiation in the equation (30) and letting s $\to 0+$. Let ρ_1 be the utilization factor of class 3 packets. Moreover, let $\boldsymbol{\Theta}_1$ denote the event that there are more than or equal to one packet of class 3 at the start of a cycle. We then have

$$\rho_1 = Pr[\boldsymbol{\Theta}_1]\frac{\overline{G}}{\overline{F}} = \frac{\overline{G}\sum_{i=1}^{N_3}\nu_i}{\overline{F}} = \frac{\overline{G}(1-\nu_0)}{\overline{F}}. \tag{31}$$

On the other hand, let $\lambda^{(3)'}$ denote the mean arrival rate of class 3 packets accepted to the system. By Little's law, we have

$$\rho_1 = \lambda^{(3)'}\overline{G}. \tag{32}$$

It follows from (31) and (32) that

$$\lambda^{(3)'} = \frac{1-\nu_0}{\overline{F}}. \tag{33}$$

Moreover, since $\lambda^{(3)'}$ satisfies $\lambda^{(3)'} = \sum_{i=0}^{N_3-1} \lambda_i^{(3)}\pi_i$, by transforming (29) we immediately have

$$\xi_j \sum_{j=0}^{N_3-1} \lambda_j^{(3)}\pi_j = \lambda_j^{(3)}\pi_j, \quad j = 0,\ldots,N_3-1. \tag{34}$$

It finally follows from (34) that

$$\pi_j = \frac{1-\nu_0}{\lambda_j^{(3)}\overline{F}}\xi_j, \quad j = 0,\ldots,N_3-1. \tag{35}$$

Furthermore, by the law of total probability, we then have

$$\pi_{N_3} = 1 - \frac{1-\nu_0}{\overline{F}} \sum_{i=0}^{N_3-1} \frac{\xi_i}{\lambda_i^{(3)}}. \tag{36}$$

Consequently we calculate $\boldsymbol{\pi}$ from (35) and (36). From these analytic results, we obtain a quantitative description of performance measures. Class 3 throughput is given by (33). Also, we calculate the mean queue length and the mean waiting time from (35) and (36), and the loss probability from (28). Moreover, we can calculate class 2 performance by setting class 2 arrival rate to 0. Further, we can calculate class 1 performance by setting class 1 and 2 arrival rates to 0.

5 Numerical results

In this subsection, we suppose that a service provider (e.g., ISP, carrier) implements the Differentiated Service. The service provider wants to realize discrimination on mean waiting time by using the Differentiated Service router modeled previously. For this purpose, we examine performance of each class, e.g., throughput, mean waiting time, mean queue length and loss probability. Finally, we conclude discrimination on mean waiting time the service provider can provide by utilizing this router.

5.1 Scenario

In the case that a service provider implements the Differentiated Service, as a matter of fact, the service provider makes a contract with each user. The tag of a packet transmitted by each user is set to *In* if the packet does not exceed the value of the contract bandwidth. Otherwise, the tag is set to *Out*.

On the other hand, based on the contract with each user, a service provider need to guarantee the total contract bandwidth. In other words, the service provider necessarily guarantees the total contract bandwidth in any situation. By guaranteeing the contract bandwidth in each class, the service provider makes discrimination of average performance, (e.g., average waiting time), between classes. However, as the DiffServ provides a QoS discrimination not for per-flow but for aggregated flows, there is nothing to be referred in detail to each flow within a class.

In this thesis, we consider the implementation of the Differentiated Service in the position of a service provider. Here, we assume that the service provider decides beforehand the total contract bandwidth of each class. We consider a scheme which guarantees the application bandwidth of each class under any overloaded situation.

When each class is stable and each buffer size is infinite, packets of each class experiences the contracted throughput without loss of packets. In our model, the assumption of finite queues leads to deteriorate each class throughput because of loss. However, the service provider need to guarantee the total contract bandwidth of each class according to the contracts. So, the service provider aims to make up for the loss by means of accepting *Out*-packets over the contract bandwidth. To the contrary, it is not desirable to accept excessive *Out*-packets with higher priority because it causes a serious impact on performance of low classes, e.g., deterioration of throughput, extension of delay. As a result, the service provider probably cannot offer QoS guarantees decided beforehand. Therefore, we assume that the service provider makes preparation for the additional admissive capacity of *Out*-packets of each class only in order to make up for the dropped *In*-packets. Note here that the decision on performance parameters (e.g., the total contract bandwidth and the additional admissible capacity) is still left to the service provider.

Besides, we consider carefully a buffer size of each class to discriminate of average waiting time. It is not very profitable to enlarge a buffer size only for the purpose of decreasing the loss probability. The reason is that the enlargement leads to the increase of waiting time. Therefore, the service provider decides the buffer size of each class with care in terms of a clear criterion. So, we assume that the service provider decides a buffer size by a criterion on *In*-packets loss probability as mentioned below.

Based upon the discussion above, we consider the appropriate decision on a buffer size by the proposed criterion. Moreover, we propose an active buffer management of traffic streams. By utilizing this management, we explain a simple scheme whose implementation leads to the desirable QoS discrimination. Finally, we conclude what discrimination of QoS guarantees a provider can offer by means of utilizing the router modeled in this thesis.

5.2 Decision on each buffer size

In implementing the Differentiated Service, a service provider need to decide in detail the contract bandwidth of each class. When the contract bandwidths of each class are, $\lambda_{In}^{(1)} = 0.100$, $\lambda_{In}^{(2)} = 0.250$ and $\lambda_{In}^{(3)} = 0.400$, respectively. we first calculate performance in the case of infinite queues (Table 1). Next, we examine each class performance (e.g., throughput ($\mathrm{Tp}^{(i)}$), average waiting time ($\mathrm{EW}^{(i)}$), average queue length ($\mathrm{EL}^{(i)}$) and loss probability ($\mathrm{LP}^{(i)}$) as a function of the buffer size N_i ($i = 1, 2, 3$) (Table 2, 3 and 4). From the results, we propose that each buffer size be decided by the criterion that loss probability of each class is less than the value on which the service provider compromises.

Table 1: Performance in infinite queues.

Class	λ_{In}	Mean Waiting Time	Mean Queue Length
1	0.100	0.555555555	0.15555555
2	0.250	0.854700853	0.46367521
3	0.400	3.076923076	1.63076923

Table 2: Class 1 performance vs. N_1
($b = 1$, $\lambda_{In}^{(1)} = 0.100$ and $\lambda_{Out}^{(1)} = 0.0$).

N_1	$\mathrm{Tp}^{(1)}$	$\mathrm{EW}^{(1)}$	$\mathrm{EL}^{(1)}$	$\mathrm{LP}^{(1)}$
1	0.0868935658	0.5083319447	0.1310643412	0.1310643412
2	0.0989771430	0.5437094919	0.1527919551	0.0102285699
3	0.0999426551	0.5543395672	0.1553448232	0.0005734492
4	0.0999974455	0.5554750862	0.1555435351	0.0000255454
5	0.0999999036	0.5555514950	0.1555549995	0.0000009643
6	0.0999999968	0.5555553849	0.1555555335	0.0000000323
7	0.0999999999	0.5555555492	0.1555555548	0.0000000010
8	0.1000000000	0.5555555553	0.1555555555	0.0000000000

Table 3: Class 2 performance vs. N_2
($b = 1$, $\lambda_{In}^{(1)} = 0.100$, $N_1 = 4$, $\lambda_{In}^{(2)} = 0.250$ and $\lambda_{Out}^{(1)} = \lambda_{Out}^{(2)} = 0.0$).

N_2	$\mathrm{Tp}^{(2)}$	$\mathrm{EW}^{(2)}$	$\mathrm{EL}^{(2)}$	$\mathrm{LP}^{(2)}$
1	0.1773712202	0.6378932207	0.2905151191	0.2905151191
2	0.2345615550	0.7412017545	0.4084189911	0.0617537800
3	0.2472185327	0.8195138177	0.4498175363	0.0111258692
4	0.2495444348	0.8462694194	0.4607262588	0.0018222607
5	0.2499283762	0.8529383151	0.4631018642	0.0002864954
6	0.2499888578	0.8543351660	0.4635631301	0.0000445688
7	0.2499982581	0.8546354592	0.4636556342	0.0000069677
8	0.2499997264	0.8546880055	0.4636714939	0.0000010944
9	0.2499999568	0.8546976730	0.4636743381	0.0000001727

Table 4: Class 3 performance vs. N_3
($b = 1,\ \lambda_{In}^{(1)} = 0.100,\ N_1 = 4,\ \lambda_{In}^{(2)} = 0.250,\ N_2 = 6,$
$\lambda_{In}^{(3)} = 0.400$ and $\lambda_{Out}^{(1)} = \lambda_{Out}^{(2)} = \lambda_{Out}^{(3)} = 0.0$).

N_3	$\mathrm{Tp}^{(3)}$	$\mathrm{EW}^{(3)}$	$\mathrm{EL}^{(3)}$	$\mathrm{LP}^{(3)}$
1	0.2133790652	1.1864953654	0.4665523371	0.4665523371
2	0.3163100409	1.5340139238	0.8015340479	0.2092248978
3	0.3589752739	1.9454105644	1.0573295640	0.1025618153
4	0.3784717143	2.2797525803	1.2412935817	0.0538207141
5	0.3882353366	2.5285356269	1.3699022168	0.0294116585
6	0.3934242752	2.7066857196	1.4583001428	0.0164393119
7	0.3962781652	2.8309495618	1.5181216634	0.0093045869
8	0.3978783168	2.9157657145	1.5579982715	0.0053042080
9	0.3987854086	2.9725922666	1.5842118304	0.0030364784
10	0.3993029253	3.0100666905	1.6012313602	0.0017426867
11	0.3995993134	3.0344476183	1.6121624984	0.0010017164
12	0.3997694557	3.0501284841	1.6191176595	0.0005763608
13	0.3998672682	3.0601153627	1.6235072387	0.0003318295
14	0.3999235511	3.0664226575	1.6262581893	0.0001911223
15	0.3999559563	3.0703774150	1.6279716914	0.0001101093
16	0.3999748080	3.0721939274	1.6287749841	0.0000629801

From Table 1, 2, 3 and 4, we see that the deterioration of throughput is inevitable because loss of *In*-packets considerably happens even though the buffer size of each class is enlarged enough. Therefore, for the service provider to decide the buffer size of each class, we suggest the following way. First, the service provider chooses the buffer size of each class where loss probability is less than a certain value. For example, when the service provider searches the buffer size where loss probability is less than $1.0\mathrm{e}^{-4}$, we choose $N_1 = 4$, $N_2 = 6$ and $N_3 = 16$ as respective buffer sizes. When we set these buffer sizes, each class has the following performance.

Table 5: Performance of each class
($b = 1,\ N_1 = 4,\ N_2 = 6,\ N_3 = 16$ and $\lambda_{Out}^{(1)} = \lambda_{Out}^{(2)} = \lambda_{Out}^{(3)} = 0.0$).

Class (Buffer Size)	λ_{In}	Throughput	Waiting Time	Loss Probability
1 (4)	0.100	0.0999974455	0.5554750862	0.0000255454
2 (6)	0.250	0.2499888578	0.8543351660	0.0000445688
3 (16)	0.400	0.3999748080	3.0721939274	0.0000629801

When these buffer sizes are obtained, we may say that it is enough to cope with the situation only *In*-packets less than the contract bandwidth flow into this router. But it is undeniable that there is a little loss of throughput.

In what follows, we consider the active buffer management of the *Out*-packets to make up for loss of *In*-packets. Note that we need avoid accepting extra *Out*-packets. In next subsection, we state the additional admissible capacity and propose the simple active buffer management which is needed to achieve the discrimination of QoS.

5.3 Additional admissible capacity and active buffer management

The additional admissible capacity is bandwidth to compensate for loss of *In*-packets. We hope that the bandwidths for each class are kept constant to the value which is almost the same as the respective target bandwidth. The reason is that to allow classes with high priority to exceed the decided bandwidth leads to the deterioration of performance of classes with lower priority. So, we consider the probabilistic acceptance of *Out*-packets to keep it constant.

In what follows, we propose the active buffer management. We suppose that *In*-packets are accepted with probability 1 regardless of on queue length. On the other hand, *Out*-packets of class i are accepted with a probability that depends on buffer occupancy of *In*-packets. In other words, *Out*-packets are accepted with probability $1-p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)})$ $(i = 1, 2, 3)$ only when *In*-packets of class i are not present in the queue. If *In*-packets are already present in the queue, the *Out*-packets are dropped with probability 1. Formally,

$$\begin{aligned} p_{In}^{(i)}(n_i) &= 0, & n_i &= 0, \ldots, N_i, \\ p_{Out}^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)}, n_i) &= \begin{cases} p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)}), & n_i = 0, \\ 1, & n_i = 1, \ldots, N_i. \end{cases} \end{aligned}$$

for i=1,2 and 3. See the lefthand side in Figure 4. On the righthand side of Figure 4, we show the total probability that arriving packets are accepted.

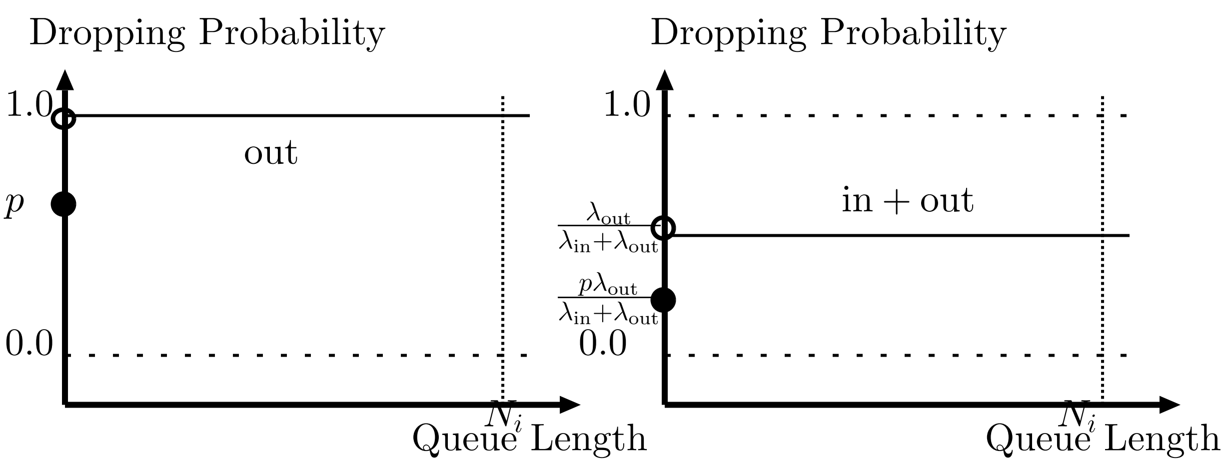

Figure 4: Active buffer management.

In what follows, we consider dropping probability $p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)})$. When each buffer is empty, the total amount of accepted packets is $\lambda_{In}^{(i)} + \Big(1 - p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)})\Big)\lambda_{Out}^{(i)}$ for $i = 1, 2$ and 3. Let $\pi_0^{(i)}$ be the probability that there is no packets of class i under the steady state. Moreover, let $\pi_{N_i}^{(i)}$ be the probability that there are N_i packets of class i under the steady state. Then, we want to have $p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)})$ such that

$$\left(1 - p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)})\right) \lambda_{Out}^{(i)} \pi_0^{(i)} + \lambda_{In}^{(i)}(1 - \pi_{N_i}^{(i)}) = \lambda_{In}^{(i)}. \tag{37}$$

By transforming (37), we have

$$\left(1 - p^{(i)}(\lambda_{In}^{(i)}, \lambda_{Out}^{(i)})\right) \lambda_{Out}^{(i)} \pi_0^{(i)} = \lambda_{In}^{(i)} \pi_{N_i}^{(i)}. \tag{38}$$

$\lambda_{In}^{(i)} \pi_{N_i}^{(i)}$ implies loss of *In*-packets' throughput. It is difficult to obtain $\lambda_{In}^{(i)} \pi_{N_i}^{(i)}$ analytically. Here, we propose to estimate $\lambda_{In}^{(i)} \pi_{N_i}^{(i)}$ approximately by the additional admissible capacity $\epsilon^{(i)}$ $(i = 1, 2, 3)$ as a substitute for $\lambda_{In}^{(i)} \pi_{N_i}^{(i)}$. Here, we define a new dropping function of *Out*-packets as $p^{(i)}(\lambda_{Out}^{(i)}, \epsilon^{(i)}, \pi_0^{(i)})$, where

$$\left(1 - p^{(i)}(\lambda_{Out}^{(i)}, \epsilon^{(i)}, \pi_0^{(i)})\right) \lambda_{Out}^{(i)} \pi_0^{(i)} = \epsilon^{(i)}. \tag{39}$$

By transforming (39), we have the following estimation formula on dropping probability of *Out*-packets:

$$p^{(i)}(\lambda_{Out}^{(i)}, \epsilon^{(i)}, \pi_0^{(i)}) = 1 - \frac{\epsilon^{(i)}}{\lambda_{Out}^{(i)} \pi_0^{(i)}}, \quad i = 1, 2, 3. \tag{40}$$

To obtain $\pi_0^{(i)}$, we approximately regard queues in this model as infinite queues. Note here that the buffer size is sufficiently large and we can disregard loss of packets. Let $\tilde{\pi}_0^{(i)}$ $(i = 1, 2, 3)$ be the approximate probability that there are not present class i packets in infinite queues. Therefore, we can decide loss probability $p^{(i)}(\lambda_{Out}^{(i)}, \epsilon^{(i)}, \pi_0^{(i)})$ from (40) if we obtain $\tilde{\pi}_0^{(i)}$ and calculate $\epsilon^{(i)}$ after we decide the respective buffer sizes.

5.3.1 Approximate formula for $\tilde{\pi}_0^{(1)}$

When class 1 queue is empty, a packet with a lower priority is being served. Suppose the event that during this service there is at least one class 1 arrival. We approximate the probability of this event by $1 - e^{-\lambda_{In}^{(1)} S_v}$, where S_v denotes the mean service time. Then, let U_1 be the average number of class 1 packet arriving during this service. We then have

$$U_1 = \frac{\sum_{k=1}^{\infty} k e^{-\lambda_{In}^{(1)} S_v} \frac{(\lambda_{In}^{(1)} S_v)^k}{k!}}{1 - e^{-\lambda_{In}^{(1)} S_v}} = \frac{\rho_1}{1 - e^{-\rho_1}},$$

where $\rho_1 = \lambda_{In}^{(1)} S_v$. Moreover, we have the average number of packets of class 1 served during a class 1 busy period,

$$U_1 + U_1 \rho_1 + U_1 \rho^2 + \cdots = \frac{U_1}{1 - \rho_1}.$$

Therefore, we have the following,

$$\pi_0^{(1)} = \frac{1 - \rho_1}{\rho_1} \frac{1 - e^{-\rho_1}}{1}. \tag{41}$$

5.3.2 Approximate formula for $\tilde{\pi}_0^{(2)}$ and $\tilde{\pi}_0^{(3)}$

Next we derive the probability that there are no class 2 packets in the infinite model. We call the sum of a service of class 2 and the following class 1 busy period a pseudo-service of class 2. Also, we call the sum of a vacation and the following class 1 busy period a pseudo-vacation of class 2. There are two kinds of class 2 busy periods.

Case (1): Busy periods starting with class 2 packets arriving in pseudo-vacations.

Case (2): Busy periods starting with class 2 packets arriving in class 1 busy periods followed by class 2 services.

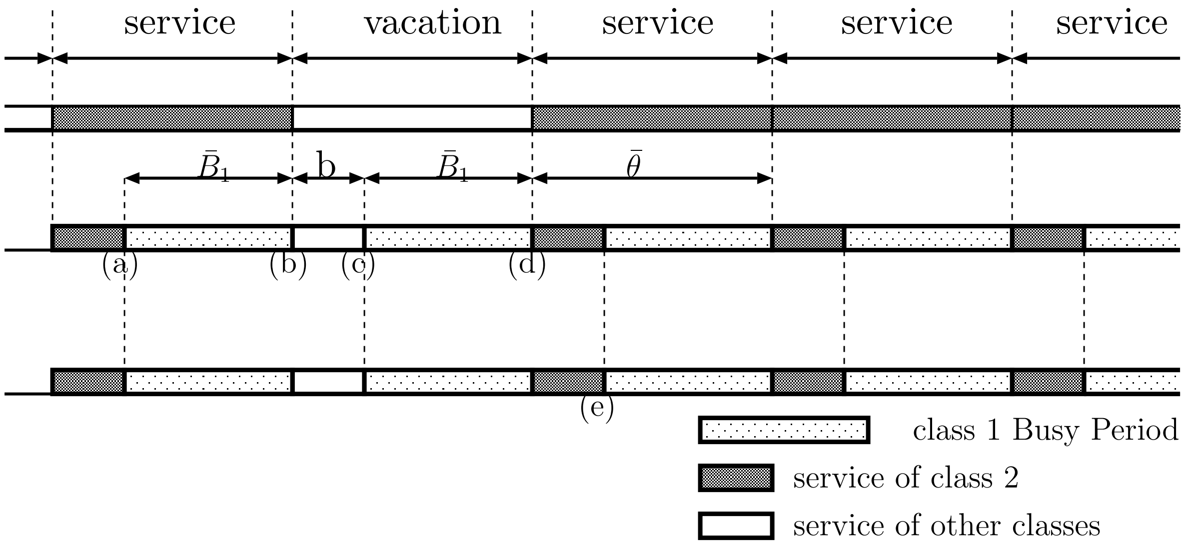

Figure 5: Embedded Markov chain in Case (1).

Let $\bar{B}_1$ be the average length of class 1 busy periods. In Case (1), the probability that there is no class 2 arrival during time interval [(a), (b)) is approximately given by $e^{-\lambda_{In}^{(2)}\bar{B}_1}$ under the steady state. In that case, there are neither class 1 nor 2 packets in the queue at time epoch (b). At time epoch (c), the average number of class 1 packets of in the system is given by ρ_1. The average busy period is given by $\bar{B}_1 = \rho_1 S_v/(1-\rho_1)$. Moreover, let $\bar{\theta}$ be the average length of pseudo-service times of class 2, which is given by $\bar{\theta} = S_v/(1-\rho_1)$.

We consider the event that there is at least one class 2 arrival during a new service of class 2. The probability of this event is approximately given by $1 - e^{-\lambda_{In}^{(2)}\bar{\theta}}$. Let $U_2^{(1)}$ be the average number of class 2 packets, at time epoch (d) given this event. Then, $U_2^{(1)}$ is given by

$$U_2^{(1)} = \frac{\lambda_{In}^{(2)}\bar{\theta}}{1 - e^{-\lambda_{In}^{(2)}\bar{\theta}}} = \frac{\rho_2}{1-\rho_1}\frac{1}{(1 - e^{-\frac{\rho_2}{1-\rho_1}})},$$

where $\rho_2 = \lambda_{In}^{(1)} S_v$. Further, we define $U_2^{(1)'}$ as the average number of class 2 packets at time epoch (e), which is given by

$$U_2^{(1)'} = U_2^{(1)} + \rho_2 - 1.$$

The average number of class 2 packets that are served successively after the epoch (e) is given by

$$U_2^{(1)'} + \lambda_{In}^{(2)} U_2' \bar{\theta} + \lambda_{In}^{(2)} \lambda_{In}^{(2)} U_2' \bar{\theta}\bar{\theta} + \cdots = \frac{U_2^{(1)'}}{1 - \lambda_{In}^{(2)} \bar{\theta}},$$

where does not include the packet initiating the busy period. Therefore, we see the average number of packets that are served during a class 2 busy period is given by

$$\frac{U_2^{(1)'}}{1 - \lambda_{In}^{(2)} \bar{\theta}} + 1 = \frac{\rho_2}{1 - \rho_1 - \rho_2} \left(\frac{1 - (1 - e^{-\frac{\rho_2}{1-\rho_1}})\rho_1}{1 - e^{-\frac{\rho_2}{1-\rho_1}}} \right). \tag{42}$$

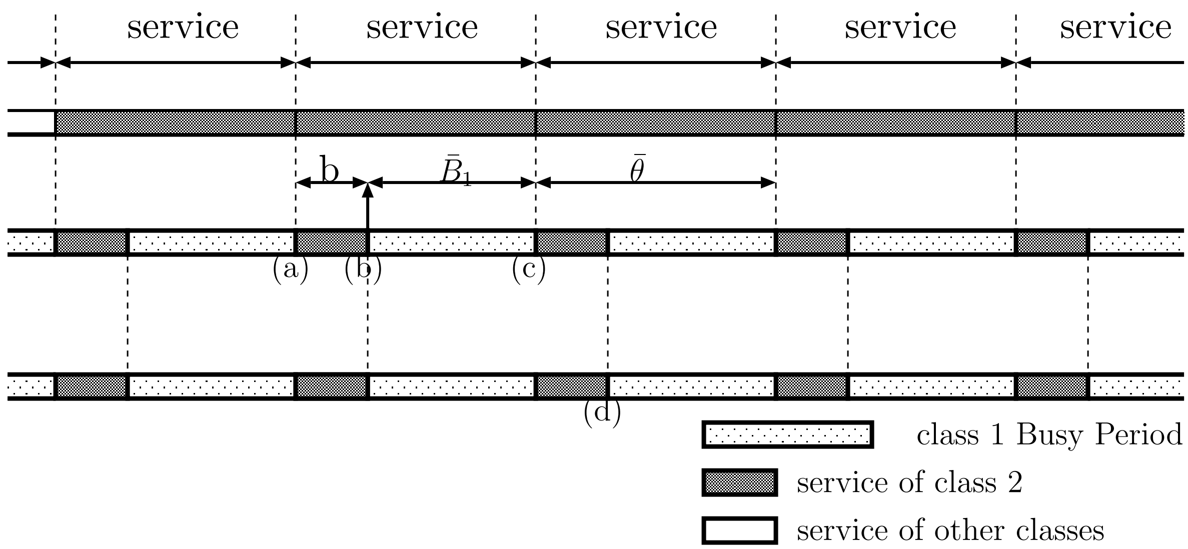

Figure 6: Embedded Markov chain in Case (2).

Next, we consider Case (2). The average probability that Case (2) happens is approximately given by $(1 - e^{-\lambda_{In}^{(2)} \bar{B}_1})$. Suppose that there is a class 2 packet at time epoch (b) and there is at least one class 2 arrival during the time interval [(b),(c)). Let $U_2^{(2)}$ be the average number of class 2 packets given this event. Then, $U_2^{(2)}$ is given by

$$U_2^{(2)} = \frac{\lambda_{In}^{(2)} \bar{B}_1}{1 - e^{-\lambda_{In}^{(2)} \bar{B}_1}}.$$

Moreover, let $U_2^{(2)'}$ be the average number of class 2 packet at time epoch (d), which is given by

$$U_2^{(2)'} = U_2^{(2)} + \rho_2 - 1$$

In Case (2), the average number of class 2 packets that are served in all during a class 2 busy period is given by

$$\frac{U_2^{(2)'}}{1 - \lambda_{In}^{(2)} \bar{\theta}} + 1 = \frac{\rho_1 \rho_2}{(1 - \rho_1 - \rho_2)} \frac{e^{-\frac{\rho_1 \rho_2}{1-\rho_1}}}{(1 - e^{-\frac{\rho_1 \rho_2}{1-\rho_1}})}. \tag{43}$$

We define $\bar{U}_2$ the average number of class 2 packets that are served during an arbitrary class 2 busy period. It then follows from (42) and (43) that

$$\begin{aligned}\bar{U}_2 &= e^{-\lambda^{(2)}_{In}\bar{B}_1}\left[\frac{\rho_2}{1-\rho_1-\rho_2}\frac{1-(1-e^{-\frac{\rho_2}{1-\rho_1}})\rho_1}{1-e^{-\frac{\rho_2}{1-\rho_1}}}\right] \\ &\quad + \quad (1-e^{-\lambda^{(2)}_{In}\bar{B}_1})\left[\frac{\rho_1\rho_2}{(1-\rho_1-\rho_2)}\frac{e^{-\frac{\rho_1\rho_2}{1-\rho_1}}}{(1-e^{-\frac{\rho_1\rho_2}{1-\rho_1}})}\right] \\ &= \frac{\rho_2}{1-\rho_1-\rho_2}\frac{e^{-\frac{\rho_1\rho_2}{1-\rho_1}}}{1-e^{-\frac{\rho_2}{1-\rho_1}}}.\end{aligned}$$

Therefore, we obtain the packet-average probability $\pi^{(2)}_0$ that an arriving packet sees the state that class 2 queue is empty, which is given by

$$\tilde{\pi}^{(2)}_0 = \frac{1}{\bar{U}_2} = \frac{1-\rho_1-\rho_2}{\rho_2}\frac{1-e^{-\frac{\rho_2}{1-\rho_1}}}{e^{-\frac{\rho_1\rho_2}{1-\rho_1}}}. \tag{44}$$

Next, we derive the probability that class 3 queue is empty. Since we now consider each queue as infinite, we can regard the sum of class 1 arrivals and class 2 arrivals as temporary class 1 arrivals and class 3 arrivals as class 2. In other words, we consider ρ_1 to be $\rho_1+\rho_2$ and ρ_3 to be ρ_2. Substituting those into (44) yields in the same discussion

$$\tilde{\pi}^{(3)}_0 = \frac{1-\rho_1-\rho_2-\rho_3}{\rho_3}\frac{1-e^{-\frac{\rho_3}{1-\rho_1-\rho_2}}}{e^{-\frac{(\rho_1+\rho_2)\rho_3}{1-\rho_1-\rho_2}}}. \tag{45}$$

Therefore, from (41), (44) and (45) we can calculate the probability that each class is empty on arrival in the steady state.

5.4 Performance of each class

We examine class 1 performance in the following three cases.

Case (1): Without the active buffer management.

Case (2): With the active buffer management when $\epsilon^{(1)} = 2.5545 \times 10^{-6}$.

Note that the value of $\epsilon^{(1)}$ was set to the loss rate of *In*-packets of class 1, which will be compensated by *Out*-packets. In both cases, service times S_v are constant and equal to one, $\lambda^{(1)}_{In} = 0.100$ and class 1 buffer size $N_1 = 4$. Table 6 shows class 1 performance in these two cases.

Table 6: Class 1 performance vs. $\lambda_{In}^{(1)} + \lambda_{Out}^{(1)}$ ($S_v = 1,\ \lambda_{In}^{(1)} = 0.100$ and $N_1 = 4$).

	Case (1)		Case (2)	
$\lambda_{\text{in+out}}^{(1)}$	$\text{Tp}^{(1)}$	$\text{EW}^{(1)}$	$\text{Tp}^{(1)}$	$\text{EW}^{(1)}$
0.100	0.099997445	0.555475086	0.099997445	0.555475086
0.150	0.149980712	0.587806578	0.100000000	0.555475273
0.200	0.199919234	0.623569629	0.100000000	0.555475273
0.300	0.299396711	0.706135995	0.100000000	0.555475273
0.400	0.397520098	0.803899866	0.100000000	0.555475273
0.600	0.582785149	1.040806013	0.100000000	0.555475273
0.800	0.738190935	1.312238437	0.100000000	0.555475273
1.000	0.850423760	1.583906993	0.100000000	0.555475273

We find that in Case (1) (i.e., without the active buffer management), it is difficult to control class 1 throughput. On the other hand, in Case (3) we observe that class 1 throughput is controlled just to the value of the contract bandwidth, i.e., $\lambda_{In}^{(1)} = 0.10$ by using the proposed active buffer management scheme.

Next, We examine class 2 performance in the following three cases.

Case (1): Without the active buffer management.

Case (2): With the active buffer management when $\epsilon^{(2)} = 1.2 \times 10^{-5}$.

In both cases, S_v is 1, $\lambda_{In}^{(1)} = 0.100$, $N_1 = 4$, $\epsilon^{(1)} = 2.5545 \times 10^{-6}$, $\lambda_{In}^{(2)} = 0.250$ and class 2 buffer size $N_2 = 6$. Table 7 shows class 2 performance in these two cases.

Table 7: Class 2 performance vs. $\lambda_{In}^{(2)} + \lambda_{Out}^{(2)}$
($S_v = 1,\ \lambda_{In}^{(1)} = 0.100,\ N_1 = 4,\ \epsilon^{(1)} = 2.5545 \times 10^{-6},\ \lambda_{In}^{(2)} = 0.250$ and $N_2 = 6$).

	Case (1)		Case (2)	
$\lambda_{\text{in+out}}^{(2)}$	$\text{Tp}^{(2)}$	$\text{EW}^{(2)}$	$\text{Tp}^{(2)}$	$\text{EW}^{(2)}$
0.250	0.249988857	0.854340240	0.249988857	0.854340240
0.300	0.299957751	0.924734135	0.250000668	0.854341588
0.400	0.399648472	1.102309769	0.250000668	0.854341588
0.500	0.498181745	1.343350496	0.250000668	0.854341588
0.600	0.593210956	1.661573935	0.250000668	0.854341588
0.700	0.680329902	2.056796821	0.250000668	0.854341588
0.800	0.753912619	2.506440087	0.250000668	0.854341588
0.900	0.809819959	2.969268443	0.250000668	0.854341588

We see that the numerical results are qualitatively very similar to those for class 1. We find that in Case (1), class 2 throughput is not well control, and it is less than the target value. On the other hand, we observe that in Case (2), class 2 throughput is well controlled to the value close to the target value, but class 2 throughput exceeded a little the contract bandwidth. The reason is that we approximately estimate the loss of class 2 throughput by the additional admissible capacity and $\pi_0^{(2)}$, $\pi_0^{(3)}$ by $\tilde{\pi}_0^{(2)}$, $\tilde{\pi}_0^{(2)}$ respectively.

Finally, We examine class 3 performance in the following two cases.

Case (1): Without the active buffer management.

Case (2): With the active buffer management when $\epsilon^{(3)} = 0.3 \times 10^{-5}$.

In both cases, S_v is 1, $\lambda_{In}^{(1)} = 0.100$, $N_1 = 4$, $\epsilon^{(1)} = 2.5545 \times 10^{-6}$, $\lambda_{In}^{(2)} = 0.250$, $N_2 = 6$, $\epsilon^{(2)} = 1.2 \times 10^{-5}$, $\lambda_{In}^{(3)} = 0.400$ and $N_3 = 16$. Table 8 shows class 3 performance in these two cases.

Table 8: Class 3 performance vs. $\lambda_{In}^{(3)} + \lambda_{Out}^{(3)}$
($S_v = 1,\ \lambda_{In}^{(1)} = 0.100,\ N_1 = 4,\ \epsilon^{(1)} = 2.5545 \times 10^{-6},\ \lambda_{In}^{(2)} = 0.250,$
$N_2 = 6,\ \epsilon^{(2)} = 1.2 \times 10^{-5},\ \lambda_{In}^{(3)} = 0.400$ and $N_3 = 16$).

	Case (1)		Case (2)	
$\lambda_{\text{in+out}}^{(3)}$	$\text{Tp}^{(3)}$	$\text{EW}^{(3)}$	$\text{Tp}^{(3)}$	$\text{EW}^{(3)}$
0.400	0.399974799	3.072386491	0.399974799	3.072386491
0.450	0.449841497	3.818028778	0.400000517	3.072387040
0.500	0.499178017	4.956989460	0.400000517	3.072387040
0.550	0.546511526	6.702469830	0.400000517	3.072387040
0.600	0.588216613	9.185123817	0.400000517	3.072387040
0.650	0.619181465	12.171764173	0.400000517	3.072387040

Again, we obtained the numerical results very similar to those we discussed. When traffic with higher priority is well controlled by the active buffer management, we can control class 3 performance. However, since class 3 performance is greatly sensitive to higher classes performance, we need to decide the each additional admissible capacity with care.

6 Conclusion

We have considered the investigation of QoS discrimination. First, we have proposed an analytic model which implements the Differentiated Service with the ideas of the AS and the PS scheme, and have obtained expressions for performance measures in a quantitative way that characterizes quality of service provided to each class. Though the model is very simple, it helps examine to offer the discrimination of QoS. To tackle important questions for service providers, we have used the priority and the active buffer management under our scenario, and proposed the discrimination scheme on performance (e.g., average waiting time).

In particular, we have achieved this discrimination by appropriate decision of each class buffer size, and by keeping each class throughput constant. So, we can control QoS in terms of the proposed scheme. This is clearly an important issue when providing a differentiated service.

Acknowledgments

The author is deeply indebted to Professor Masao Fukushima for his encouragement. The author would sincerely like to express his thanks to Associate Professor Tetsuya Takine for his continual guidance and valuable suggestions to accomplish this thesis. The author is also grateful to them for their careful reading of this manuscript and accurate comments.

References

[1] Braden, R., Clark, D. and Shenker, S., "Integrated Services in the Internet Architecture: an Overview," RFC 1633, June 1944.

[2] Cris, M., "IP QoS : Traveling in First Class on the Internet," *IEEE Internet Computing*, vol. 3, no. 2, pp. 84–88, March/April 1999.

[3] Zhang, L., Estrin, D., Deering, S., Shenker, S. and Zappala, D., "RSVP : A New Resource ReSerVation Protocol," *IEEE Network*, vol. 5, pp. 8–18, September 1993.

[4] Braden, R., Zhang, L., Berson, S., Herzog, S. and Jamin, S., "Resource ReSerVation Protocol (RSVP) – Version 1 Functional Specification," RFC 2205, September 1997.

[5] Floyd, S. and Jacobson, V., "Random Early Detection Gateways for Congestion Avoidance," *IEEE/ACM Transactions on Networking*, vol. 1, no. 4, pp. 397–413, August 1993.

[6] Nichols, K., Blake, S., Baker, F. and Black, D., "Definition of the Differentiated Services Field (DS Field) in the IPv4 and IPv6 Headers," RFC 2474, December 1997.

[7] Blake, S., Black, D., Carlson, M., Davies, E., Wang, Z. and Weiss, W., "An Architecuture for Differentiated Services," RFC 2475, December 1998

[8] Nichols, K., Jacobson, V. and Zhang, L., "A Two-Bit Differentiated Services Architecture for the Internet," RFC 2638, July 1999.

[9] Clark, D., "Adding Service Discrimination to the Internet," LCS MIT Technical report, September 1993.

[10] Heinanen, J., Baker, F., Weiss, W. and Wroclawski, J., "Assured Forwording PHB Group," RFC 2597, June 1999.

[11] Jacobson, V., Nichols, K. and Poduri, K., "An Expedited Forwording PHB," RFC 2598, July 1999.

[12] Heyman, D., "A priority queueing system with server interference," *SIAM Journal Applied Mathematics*, vol. 17, no. 1, pp. 74–82, January 1969.

[13] Courtois, P., "The M/G/1 finite capacity queue with delays," *IEEE Transactions on Communications*, vol. Com-28, no. 2, pp. 165–172, February 1980.

[14] Kramer, M., "Wartesysteme M/G/1 beschrãnkter Kapazitã mit sperrphasen un relativen Prioritãten," *Informatik-Fachberichte* 61, Spriger-Verlag, no. 61, pp. 150–164, 1983.

[15] May, M., Bolot, J. and Diot, C., "Simple Perforemance Models of Differentiated Services Schemes for the Internet," *IEEE INFOCOM '99*, pp. 1385–1394, NY, 1999.