Optimal Design of a Combined Heat and Power Network

Guidance

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February 2010

Abstract

In recent years, our society is under pressure to make effective use of the energy because of global warming and high energy prices. As a means of solving the problem, combined heat and power (CHP) systems, which make efficient use of the heat produced as a by-product of electricity generation, and microgrids, in which surplus electricity can be traded among the users, have been attracting a great deal of attention. In this paper, we propose a combined heat and power network model in which surplus electricity and heat can be transmitted among the nodes in the network. We formulate the model as an optimization problem in which the cost of the whole network is minimized under the condition that the given energy demands are satisfied at all nodes. Next, from the game theoretic viewpoint, we formulate the problem as a generalized Nash equilibrium problem (GNEP) in which each node minimizes its own cost. We show that an optimal solution of the weighted total cost minimization problem is actually equal to a normalized equilibrium of the GNEP. Finally, we make some numerical experiments and discuss the obtained results.

Contents

1 Introduction

In recent years, global climate change caused by the warming of the earth has become a serious issue. Some countries have set up a greenhouse gas reduction goal and keep trying to reduce greenhouse gas such as CO2 to cope with global warming. Japan also has pledged to reduce by 6 percent from its 1990 emissions by 2012 under the Kyoto Protocol. One of the effective actions is to make efficient use of energy resources and to reduce consumption of energy. In addition to global environmental problems, high energy prices strengthen the importance of effective energy utilization.

The system which produces electricity by power-generating equipments, such as an engine and a fuel battery, placed near a point of demand, and makes use of simultaneously generated heat is called a combined heat and power (CHP) system [8, 9, 10]. The advantage of this system is that it can use effectively heat energy, which would be emitted into the air in the case of conventional power plants, and reduce a loss of electric transmission.

There is a system that controls electric production according to varying demand by connecting equipments, such as CHP systems, solar batteries, wind power generator, storage batteries, with the power grid. We call such a network a "microgrid" [1, 5]. The microgrid can effectively be used to reduce the uncontrollable risk of electricity generated by natural energy, since surplus electricity can be traded as a commodity and extra electricity can be stored in batteries for future usage. In Japan, experimental research on the microgrid has been under investigation since 2003 [3, 6].

In this paper, we focus on the CHP system based on energy transmission. In particular, we consider conventionally unused heat being produced simultaneously when generating electricity. The network with transmission of electricity and heat is called the Combined Heat and Power network, or simply the CHP network. Given the energy demand at each node in the CHP network, we seek the optimal situation in which the total cost of all nodes are minimized.

On the other hand, we may consider the game in which each node minimizes its own cost, instead of the problem of minimizing the total cost at all nodes. In this game, the operating conditions at a node are affected by other nodes through energy transmission. So it can be formulated as a generalized Nash equilibrium problem (GNEP) [2], which is a generalization of the well-known Nash equilibrium problem (NEP). The NEP is to find a profile of strategies in which each player's strategy is an optimal response to the rival players' strategies. Besides, if each player's strategy set depends on the other players' strategies, this problem is called the GNEP. In general, the GNEP can have multiple, or even infinitely many solutions. In this paper, we are particularly interested in an equilibrium point called a normalized equilibrium point [7], and discuss the relation between the solutions obtained from the two formulations, the GNEP and the total cost minimization problem, for the CHP network.

The paper is organized as follows: In the next section, we describe a model of a town that consists of a number of nodes. In particular, we explain the characteristics of equipments installed at each node and the energy transmission among nodes. Moreover we give the definition of energy costs at each node and formulate the total energy cost minimization problem under the condition that all energy demands are satisfied. In Section 3, from the game theoretic viewpoint, we formulate the CHP network as a GNEP and clarify the relation between the total cost minimization problem and the GNEP of finding a normalized equilibrium point. In Section 4, we report some numerical results with the CHP network model and demonstrate that the energy transmission in the network can be helpful in making effective use of generated energy. Finally we conclude with some remarks in Section 5.

2 Model formulation

In this section, we propose a CHP network model and then formulate a total cost minimization problem with energy transmission.

First, we describe the model of a town considered in this paper. The town consists of several locations labeled $L_1, \dots, L_K$. We assume that the demands for electricity and heat are given at each location. In order to satisfy their demands, they buy electricity from an electricity firm, or purchase gas from a gas company and make heat from gas. In addition, we suppose that they can generate electricity and heat at their locations, by using several devices such as a combined heat and power, a solar battery and a solar heat panel. We also assume that they can buy and sell surplus electricity and heat by themselves. Furthermore, we presume that there already exist transmission lines through which electricity and heat are transmitted.

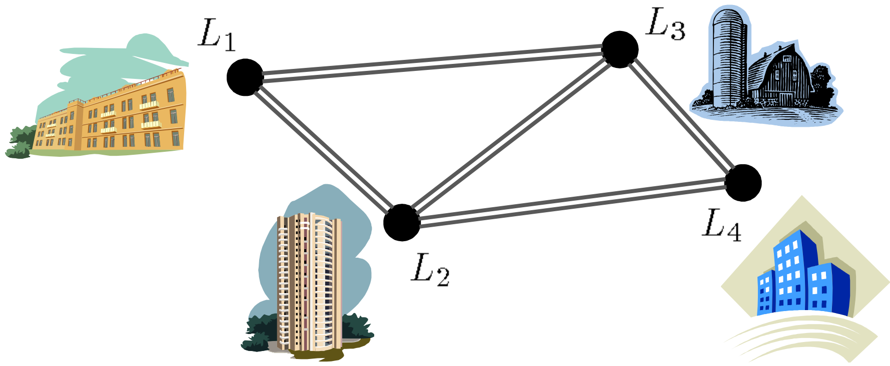

Fig. 1: Network model of a town

We represent the whole town as an undirected graph $G = (N, A)$, regarding a location as a node and viewing a transmission line as an arc, where N is a node set and A is an arc set. We denote a node corresponding to a location L_i by i, and denote a set of node i's adjacent nodes by $N(i)$. The arc set A is a set of energy transmission lines which are placed in the whole town. For example, we can represent the town shown in Fig.1 by the graph $G = (N, A)$ with $N = \{1, 2, 3, 4\}$, $A = \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)\}$, and $N(1) = \{2, 3\}$, $N(2) = \{1, 3, 4\}$, $N(3) = \{1, 2, 4\}$, $N(4) = \{2, 3\}$.

Second, we give an explanation about each node $i \in N$. Let the planning horizon be $t = 1, \dots, T$. We assume that the electricity demand $e_i(t)$, $t = 1, \dots, T$, and the heat demand $h_i(t)$, $t = 1, \cdots, T$, are given at each node, and that there are a gas engine, a gas boiler, a solar battery, a solar heat panel, a storage battery and a heat storage tank, at all nodes. By using these equipments, they can produce electricity and heat by themselves. We will explain these equipments individually in more detail later. We also assume that they can trade electricity and heat between nodes, in addition to buying electricity from an electricity firm and gas from a gas company.

We use the following notations throughout the paper. For time $t = 1, \dots, T$, $z_i(t)$ denotes the volume of electricity that node i purchases from an electricity firm, $a_{ji}(t)$ and $c_{ji}(t)$ represent the quantities of electricity and heat that node i purchases from other nodes $j \in N(i)$, respectively. Similarly, for each time $t = 1, \dots, T$, $b_{ij}(t)$ and $d_{ij}(t)$ denote the amounts of selling electricity and heat, respectively. If there is a leftover energy, it can be stored in the battery or the tank. Alternatively, surplus electricity can also be sold to an electric power company, and surplus heat is wasted. For time $t = 1, \dots, T$, $w_i(t)$ represents the amount of selling electricity to an electricity firm, and $s_i(t)$ denotes the quantity of waste heat. Fig.2 shows the energy flow at node i.

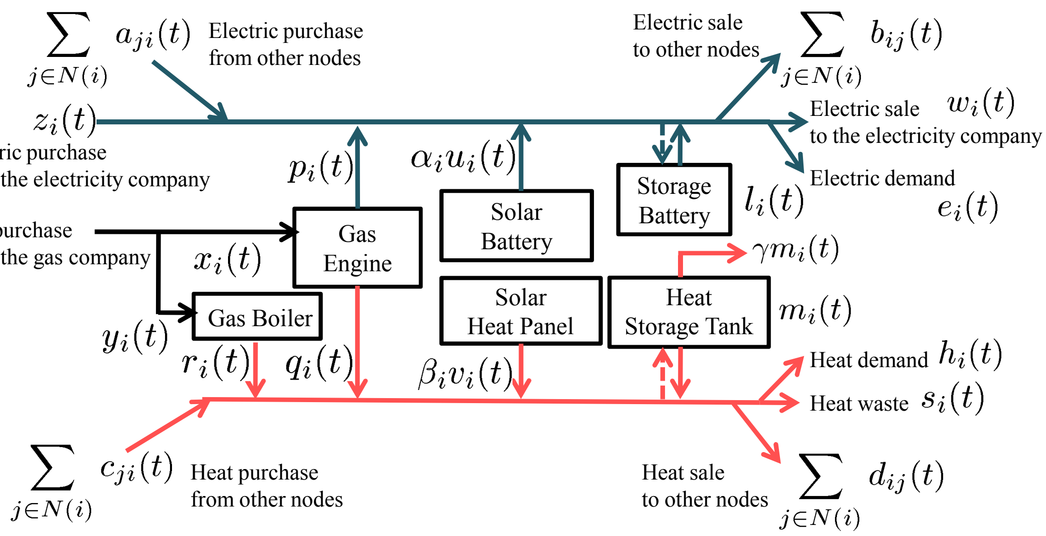

Fig. 2: Energy flow at node i

In order to satisfy the energy demand at each node, they produce the energy at their own node, and buy it from a firm and other nodes. In the following, we consider how to determine the strategy of operating equipments and the volume of energy trade to minimize the total energy cost.

Characteristics of equipments

First, we describe the characteristics of the equipments at node i.

A gas engine is a machine to generate electricity and heat by using gas. We denote the input quantity of a gas engine by $x_i(t)$, the electricity output quantity by $p_i(t)$, and the heat output quantity by $q_i(t)$. Then the relations between inputs and outputs at each time are given as follows:

$$p_i(t) = P_i x_i(t), \tag{1}$$

$$q_i(t) = Q_i x_i(t), \tag{2}$$

where P_i and Q_i are positive constants which represent the property of a gas engine. There is a maximum limitation of gas input $\bar{x}_i$, at each t,

$$x_i(t) \leq \bar{x}_i. \tag{3}$$

A gas boiler is an equipment which makes heat from gas. For time $t = 1, \ldots, T$, we have the following relation between the input amount of gas $y_i(t)$ and the heat output quantity $r_i(t)$:

$$r_i(t) = R_i y_i(t), \tag{4}$$

where R_i is a positive constant which represents the property of a gas boiler.

A solar battery and a solar heat panel are devices which convert the energy of sunlight into electricity and heat, respectively. We assume that the area of a solar battery at node i is given by α_i, and that the electric production of a solar battery per unit area is given by $u_i(t)$. We also assume that the area of a solar heat panel and the heat production of a solar heat panel per unit area at node i are given by β_i and $v_i(t)$, respectively. We suppose that α_i, u_i, β_i, and v_i are given.

A storage battery is a device where electricity can be charged or discharged whenever needed. We denote the quantity of stored electricity in a storage battery at the beginning of time t by $l_i(t)$. There is a maximum value of stored electricity $\bar{l}_i$, for each time t,

$$l_i(t) \leq \bar{l}_i, \tag{5}$$

$$l_i(1) = l_{i_0}, \tag{6}$$

where l_{i_0} represents the quantity of electricity stored at the initial time.

A heat storage tank is a device that can store heat and release heat at any time. We denote the quantity of heat stored in a heat storage tank at the beginning of time t by $m_i(t)$. Suppose that heat in a tank is lost into the atmosphere in proportion to $m_i(t)$. By using a coefficient $\gamma \in (0, 1)$, the amount of the heat loss at time t is given by

$$\gamma m_i(t).$$

We denote a maximum limitation of the heat storage tank by $\bar{m}_i$ at each time t,

$$m_i(t) \leq \bar{m}_i, \tag{7}$$

$$m_i(1) = m_{i_0}, \tag{8}$$

where m_{i_0} represents the quantity of heat stored at the initial time.

These are the characteristics of equipments at node i. Since we suppose that the same equipments are installed at all nodes $i = 1, \ldots, K$, these conditions hold for all i.

Energy balances

Next, we consider energy balances. At each node, the energy demands must be satisfied at any time.

As to the electricity, the following equality holds for each i and t:

$$z_i(t) + \sum_{j \in N(i)} a_{ji}(t) + p_i(t) + \alpha_i u_i(t) + l_i(t) = e_i(t) + w_i(t) + \sum_{j \in N(i)} b_{ij}(t) + l_i(t+1), \quad (9)$$

where the left-hand side means the amount of the electricity transmitted into node i, generated at node i, and stored in a battery at t, while the right-hand side represents the total quantity of the electricity usage, the electricity transmitted out of node i at t, and the electricity stored in a battery at $t+1$.

Similarly, the following relation about heat holds for each i and t:

$$\begin{aligned} & r_i(t) + \sum_{j \in N(i)} c_{ji}(t) + q_i(t) + \beta_i v_i(t) + m_i(t) \\ = & \; h_i(t) + s_i(t) + \sum_{j \in N(i)} d_{ij}(t) + \gamma m_i(t) + m_i(t+1), \end{aligned} \quad (10)$$

where the left-hand side equals the sum of the heat transmitted into node i, generated at node i, and stored in a tank at t, while the right-hand side represents the total amount of the heat used, the heat transmitted out of node i, and heat emitted from a tank at t, and the heat stored in a tank at $t+1$.

The relation of energy transmission among nodes

We consider the energy transmission among nodes. The amount of energy which node i gets from node j equals the amount of energy which node j transmits to node i. Hence, we have the following relation:

$$a_{ji}(t) = b_{ji}(t), \quad (i,j) \in A, \quad (11)$$

$$c_{ji}(t) = d_{ji}(t), \quad (i,j) \in A. \quad (12)$$

Energy costs

We consider the energy costs at node i in time t. The gas utility cost increases in proportion to the purchase volume, and is given by

$$C_1 \sum_{t=1}^{T} (x_i(t) + y_i(t)),$$

where C_1 denotes the gas unit price.

The electric utility cost that has to be paid to the electricity company is the sum of the contract fee depending on the contract quantity $g_i \geq 0$ and the usage charge that depends on the amount of electricity usage. The contract fee can be written as

$$C_2 g_i,$$

where C_2 is the unit contract fee of electricity. Furthermore, we suppose the unit price of electricity is C_3 when the electricity usage is less than the contract quantity, and it is C_4 when the electricity usage exceeds the contract quantity. Naturally we assume $C_3 < C_4$. Then the usage charge to be paid to the electric power company is given by

$$\max \{C_3 z_i(t), C_4 z_i(t) + (C_3 - C_4) g_i\}.$$

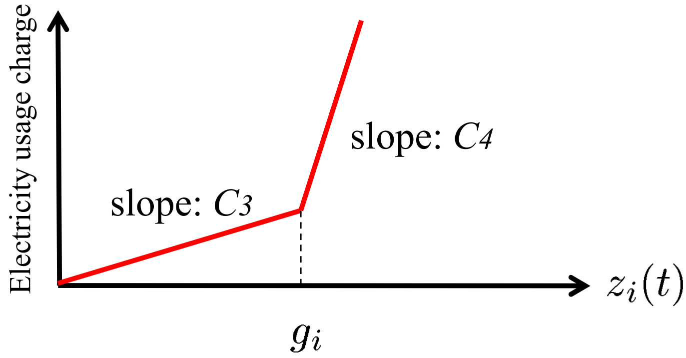

Fig. 3: Electricity utility cost

Fig 3 shows the relation between the quantities of electricity purchased from the electric power company and the electricity usage charge. Therefore, the total electric usage charge during the planning horizon is given by

$$\sum_{t=1}^{T} \max \{C_3 z_i(t), C_4 z_i(t) + (C_3 - C_4) g_i\}.$$

The cost of electricity that node i purchases from other nodes is given by

$$C_5 \sum_{t=1}^{T} \sum_{j \in N(i)} a_{ji}(t),$$

where C_5 is the unit buying price between nodes.

The profit gained by selling electricity to other nodes can be written as

$$C_6 \sum_{t=1}^{T} \sum_{j \in N(i)} b_{ij}(t),$$

where C_6 is the unit sales price between nodes.

The benefit by selling surplus electricity to the electric power company is given by

$$C_7 \sum_{t=1}^{T} w_i(t),$$

where C_7 is the unit sales price to the electric power company.

Similarly, we consider the heat trading cost among nodes. Let C_8 and C_9 be the unit purchase price and the unit sales price of heat between nodes. So, the total purchase cost in the planning horizon is given by

$$C_8 \sum_{t=1}^{T} \sum_{j \in N(i)} c_{ji}(t),$$

and the total sales cost in the planning horizon can be written as

$$C_9 \sum_{t=1}^{T} \sum_{j \in N(i)} d_{ij}(t).$$

Unsold surplus heat is discarded, and so no profit can be obtained.

We regard the profit as the negative cost. Therefore, the total cost at node i is given by

$$\begin{aligned} f_i = C_1 \sum_{t=1}^{T} (x_i(t) + y_i(t)) + C_2 g_i + \sum_{t=1}^{T} \max\{C_3 z_i(t), C_4 z_i(t) + (C_3 - C_4) g_i\} \\ + C_5 \sum_{t} \sum_{j \in N(i)} a_{ji}(t) - C_6 \sum_{t=1}^{T} \sum_{j \in N(i)} b_{ij}(t) - C_7 \sum_{t=1}^{T} w_i(t) \\ + C_8 \sum_{t=1}^{T} \sum_{j \in N(i)} c_{ji}(t) - C_9 \sum_{t=1}^{T} \sum_{j \in N(i)} d_{ij}(t). \end{aligned} \quad (13)$$

The weighted total cost minimization problem

In the rest of this section, we formulate the weighted total cost minimization problem.

The objective function is given by the sum of the energy costs at all nodes, i.e.,

$$\sum_{i=1}^{K} \omega_i f_i,$$

where f_i is given by (13) and $\omega_i > 0$, $i = 1, \cdots, K$ are weight factors which represent the relative importance of the nodes.

The constraints of this problem consist of the characteristics of the equipments (1) – (7), the energy balance (9),(10), and the relation of the energy transmission (11),(12). In addition, the variables corresponding to energy are nonnegative at each time, i.e.,

$$g_i, x_i(t), y_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t), a_{ji}(t), b_{ij}(t), c_{ji}(t), d_{ij}(t) \geq 0 \quad (14)$$

If these inequalities hold, then $p_i(t)$, $q_i(t)$, and $r_i(t)$ are all nonnegative from (1), (2), and (4).

Moreover, in order to prevent some nodes from suffering a loss by maximizing the total profit, we add the following constraint for each i:

$$f_i \leq f_i^*. \quad (15)$$

This constraint requires that the total cost f_i at each node is not greater than a given standard value f_i^* at that node.

From the above discussion, by using f_i given by (13), we can write the weighted total cost minimization problem as the follows:

$$\begin{aligned}
\text{minimize}\quad & \sum_{i=1}^{K} \omega_i f_i \\
\text{subject to}\quad & g_i, x_i(t), y_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t) \geq 0, \quad \forall t, \\
& a_{ji}(t), b_{ij}(t), c_{ji}(t), d_{ij}(t) \geq 0, \quad \forall t, \forall (i,j) \in A, \\
& p_i(t) = P_i x_i(t),\ q_i(t) = Q_i x_i(t),\ r_i(t) = R_i y_i(t), \quad \forall t, i, \\
& x_i(t) \leq \bar{x}_i,\ y_i(t) \leq \bar{y}_i,\ l_i(t) \leq \bar{l}_i,\ m_i(t) \leq \bar{m}_i,\ l_i(1) = l_{i_0},\ m_i(1) = m_{i_0}, \quad \forall t, i, \\
& z_i(t) + \sum_{j \in N(i)} a_{ji}(t) + p_i(t) + \alpha_i u_i(t) + l_i(t) \\
& \quad = e_i(t) + w_i(t) + \sum_{j \in N(i)} b_{ij}(t) + l_i(t+1), \quad \forall t, i, \\
& r_i(t) + \sum_{j \in N(i)} c_{ji}(t) + q_i(t) + \beta_i v_i(t) + m_i(t) \\
& \quad = h_i(t) + s_i(t) + \sum_{j \in N(i)} d_{ij}(t) + \gamma m_i(t) + m_i(t+1), \quad \forall t, i, \\
& a_{ji}(t) = b_{ji}(t),\ c_{ji}(t) = d_{ji}(t), \quad \forall (i,j) \in A, \\
& f_i \leq f_i^*, \quad \forall i.
\end{aligned}$$

Note that f_i contains the maximum function on the right-hand side of (13). By introducing auxiliary variables $\zeta_i(t)$, we can represent the optimization problem without using the maximum function in the objective function. By (1), (2), and (4), we can substitute $P_i x_i(t)$, $Q_i x_i(t)$, and $R_i y_i(t)$ for $p_i(t)$, $q_i(t)$, and $r_i(t)$, respectively. Moreover, we can eliminate $a_{ji}(t)$ and $c_{ji}(t)$ by using (11) and (12). Consequently, we obtain the following linear programming problem:

$$\begin{aligned}
\text{minimize}\quad & \sum_{i=1}^{K} \omega_i \Bigg\{ C_1 \sum_{t=1}^{T} (x_i(t) + y_i(t)) + C_2 g_i + \sum_{t=1}^{T} \zeta_i(t) + C_5 \sum_{t=1}^{T} \sum_{j \in N(i)} b_{ji}(t) \\
& \quad - C_6 \sum_{t=1}^{T} \sum_{j \in N(i)} b_{ij}(t) - C_7 \sum_{t=1}^{T} w_i(t) + C_8 \sum_{t=1}^{T} \sum_{j \in N(i)} d_{ji}(t) - C_9 \sum_{t=1}^{T} \sum_{j \in N(i)} d_{ij}(t) \Bigg\} \\
\text{subject to}\quad & g_i, x_i(t), y_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t), b_{ij}(t), d_{ij}(t) \geq 0, \quad \forall t, \forall (i,j) \in A, \\
& C_3 z_i(t) \leq \zeta_i(t),\ C_4 z_i(t) + (C_3 - C_4) g_i \leq \zeta_i(t), \quad \forall t, i, \\
& x_i(t) \leq \bar{x}_i,\ y_i(t) \leq \bar{y}_i,\ l_i(t) \leq \bar{l}_i,\ m_i(t) \leq \bar{m}_i,\ l_i(1) = l_{i_0},\ m_i(1) = m_{i_0}, \quad \forall t, i, \\
& z_i(t) + \sum_{j \in N(i)} b_{ji}(t) + P_i x_i(t) + \alpha_i u_i(t) + l_i(t) \\
& \quad = e_i(t) + w_i(t) + \sum_{j \in N(i)} b_{ij}(t) + l_i(t+1), \quad \forall t, i, \\
& R_i y_i(t) + \sum_{j \in N(i)} d_{ji}(t) + Q_i x_i(t) + \beta_i v_i(t) + m_i(t) \\
& \quad = h_i(t) + s_i(t) + \sum_{j \in N(i)} d_{ij}(t) + \gamma m_i(t) + m_i(t+1), \quad \forall t, i, \\
& C_1 \sum_{t=1}^{T} (x_i(t) + y_i(t)) + C_2 g_i + \sum_{t=1}^{T} \zeta_i(t) + C_5 \sum_{t=1}^{T} \sum_{j \in N(i)} b_{ji}(t) - C_6 \sum_{t=1}^{T} \sum_{j \in N(i)} b_{ij}(t) \\
& \quad - C_7 \sum_{t=1}^{T} w_i(t) + C_8 \sum_{t=1}^{T} \sum_{j \in N(i)} d_{ji}(t) - C_9 \sum_{t=1}^{T} \sum_{j \in N(i)} d_{ij}(t) \leq f_i^*, \quad \forall i.
\end{aligned}$$

The decision variables in this problem are

$$g_i, x_i(t), y_i(t), \zeta_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t), b_{ij}(t), d_{ij}(t), \quad \forall t, \forall (i,j) \in A.$$

We call this problem the weighted total cost minimization problem. When the weight factors are chosen as $\omega_i = 1$, $i = 1, \cdots, K$, we simply call this problem the total cost minimization problem.

Next, we discuss how to choose the standard values f_i^*. Maximizing the total profit may sacrifice the benefit of several nodes. If a node suffers a loss, then the node will have no incentive to participate in the network. So we choose f_i^* so that each node's cost does not increase from the one incurred when energy is not transmitted among nodes. When the volumes of energy transmission such as a_{ij}, b_{ij}, c_{ij}, and d_{ij} are equal to 0 in the total cost minimization problem, the problem reduces to the cost minimization problem with no energy transmission. So, at each node i, the schedule of operating equipments is decided by solving the following problem:

$$\begin{array}{ll} \text{minimize} & C_1 \sum_{t=1}^{T} \{x_i(t) + y_i(t)\} + C_2 g_i + \delta_i(t) - C_7 \sum_{t=1}^{T} w_i(t) \\ \text{subject to} & g_i, x_i(t), y_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t) \geq 0, \quad \forall t, \\ & C_3 z_i(t) \leq \delta_i(t),\ C_4 z_i(t) + (C_3 - C_4) g_i \leq \delta_i(t), \quad \forall t, \\ & x_i(t) \leq \bar{x}_i,\ y_i(t) \leq \bar{y}_i,\ l_i(t) \leq \bar{l}_i,\ m_i(t) \leq \bar{m}_i,\ l_i(1) = l_{i_0},\ m_i(1) = m_{i_0}, \quad \forall t, \\ & z_i(t) + P_i x_i(t) + \alpha_i u_i(t) + l_i(t) = e_i(t) + w_i(t) + l_i(t+1), \quad \forall t, \\ & R_i y_i(t) + Q_i x_i(t) + \beta_i v_i(t) + m_i(t) = h_i(t) + s_i(t) + \gamma m_i(t) + m_i(t+1), \quad \forall t. \end{array}$$

Notice that this optimization problem is independent of other nodes. The optimal value of the problem is nothing but the minimum cost when node i makes an operating schedule by itself without energy transmission. So, it is reasonable to employ the optimal value as the standard value f_i^*. As a result, the cost of each node in the optimal solution of the total cost minimization problem does not exceed the cost of that node incurred when energy is not transmitted in the network.

3 Game theoretic formulation

In this section, after recalling the basic concepts in the generalized Nash equilibrium problem (GNEP), we formulate the cost minimization problem as a GNEP, and we discuss its relation with the weighted total cost minimization problem described in the previous section.

The GNEP is to find a solution in which no player has an incentive to change his own strategy unilaterally, where each player's strategy set depends on the others' strategies. Let $\theta^i \in \Re^{N_i}$ denote player i's strategy, where N_i is a positive integer. The vector formed by all players' strategies is denoted $\theta := (\theta^i)_{i=1}^K \in \Re^N$, where $N := \sum_{i=1}^K N_i$. Moreover, the vector formed by all players' strategies except those of player i is denoted $\theta^{-i} := (\theta^{i'})_{i'=1, i' \neq i}^K \in \Re^{N-N_i}$. In the GNEP, each player solves the following minimization problem, with θ^{-i} being exogenous variables:

$$P_i(\theta^{-i}): \quad \begin{array}{ll} \text{minimize} & f_i(\theta^i, \theta^{-i}) \\ \text{subject to} & \theta^i \in S_i(\theta^{-i}), \end{array} \tag{16}$$

where $f_i : \Re^N \to \Re$ is the cost function of player i, and $S_i(\theta^{-i}) \subseteq \Re^{N_i}$ is the strategy set of player i. Note that the strategy set of player i depends on the other players' strategies.

A generalized Nash equilibrium (GNE) is defined to be a tuple $\theta^* := (\theta^{*,i})_{i=1}^K$ such that $\theta^{*,i}$ is an optimal solution of the following optimization problem for each $i = 1, \ldots, N$:

$$P_i(\theta^{*,-i}): \quad \begin{array}{ll} \text{minimize} & f_i(\theta^i, \theta^{*,-i}) \\ \text{subject to} & \theta^i \in S_i(\theta^{*,-i}). \end{array}$$

In particular, if each player's strategy set does not depend on the other players' strategies, the GNEP and GNE reduce to the Nash equilibrium problem (NEP) and the Nash equilibrium (NE), respectively.

In what follows, we assume that each player's strategy set $S_i(\theta^{-i})$ is defined by

$$\begin{aligned} S_i(\theta^{-i}) &= X_i \cap Y_i(\theta^{-i}), \\ Y_i(\theta^{-i}) &= \{\theta^i \in \Re^{N_i} \mid (\theta^i, \theta^{-i}) \in Y\}, \end{aligned}$$

where X_i is a nonempty closed convex subset of $\Re^{N_i}$ and Y is a nonempty closed convex subset of R^N, Then, each player solves the following problem:

$$P_i(\theta^{-i}): \quad \begin{array}{ll} \text{minimize} & f_i(\theta^i, \theta^{-i}) \\ \text{subject to} & \theta^i \in X_i \cap Y_i(\theta^{-i}). \end{array} \tag{17}$$

Note that the set $Y_i(\theta^{-i})$ is defined by using the set Y common to all players.

Next, we consider the CHP network model where each node put a high priority on minimizing its own cost rather than the total cost of the whole network. Then the constraints of each node are the characteristics of equipments (1) – (7), the energy balance (9),(10), the relations of energy transmission (11),(12), the energy nonnegativity constraints (14), and the cost limitation constraint of each node (15). The objective function is given by (13). So the optimization problem of node i is formulated as the following problem:

$$\begin{array}{ll} \text{minimize} & C_2 g_i + \displaystyle\sum_{t=1}^{T} \Big\{ C_1(x_i(t) + y_i(t)) + \zeta_i(t) - w_i(t) \\ & \qquad + C_5 \displaystyle\sum_{j \in N(i)} a_{ji}(t) - C_6 \sum_{j \in N(i)} b_{ij}(t) + C_8 \sum_{j \in N(i)} c_{ji}(t) - C_9 \sum_{j \in N(i)} d_{ij}(t) \Big\} \\ \text{subject to} & g_i, x_i(t), y_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t) \geq 0, \quad \forall t, \\ & a_{ji}(t), b_{ij}(t), c_{ji}(t), d_{ij}(t) \geq 0, \quad \forall t, \forall j \in N(i), \\ & C_3 z_i(t) \leq \zeta_i(t), \ C_4 z_i(t) + (C_3 - C_4) g_i \leq \zeta_i(t), \quad \forall t, \\ & x_i(t) \leq \bar{x}_i, \ y_i(t) \leq \bar{y}_i, \ l_i(t) \leq \bar{l}_i, \ m_i(t) \leq \bar{m}_i, \ l_i(1) = l_{i_0}, \ m_i(1) = m_{i_0}, \quad \forall t, \\ & z_i(t) + \displaystyle\sum_{j \in N(i)} a_{ji}(t) + P_i x_i(t) + \alpha_i u_i(t) + l_i(t) \\ & \quad = e_i(t) + w_i(t) + \displaystyle\sum_{j \in N(i)} b_{ij}(t) + l_i(t+1), \quad \forall t, \\ & r_i y_i(t) + \displaystyle\sum_{j \in N(i)} c_{ji}(t) + Q_i x_i(t) + \beta_i v_i(t) + m_i(t) \\ & \quad = h_i(t) + s_i(t) + \displaystyle\sum_{j \in N(i)} d_{ij}(t) + \gamma m_i(t) + m_i(t+1), \quad \forall t, \\ & C_2 g_i + \displaystyle\sum_{t=1}^{T} \Big\{ C_1(x_i(t) + y_i(t)) + \zeta_i(t) - w_i(t) + C_5 \sum_{j \in N(i)} a_{ji}(t) - C_6 \sum_{j \in N(i)} b_{ij}(t) \\ & \qquad + C_8 \displaystyle\sum_{j \in N(i)} c_{ji}(t) - C_9 \sum_{j \in N(i)} d_{ij}(t) \Big\} \leq f_i^*, \\ & a_{ji}(t) = b_{ji}(t), \ c_{ji}(t) = d_{ji}(t), \quad \forall (i,j) \in A. \end{array}$$

The decision variables of this problem are

$$g_i, x_i(t), y_i(t), \zeta_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t), a_{ji}(t), b_{ij}(t), c_{ji}(t), d_{ij}(t), \quad \forall t, \forall j \in N(i),$$

and the vector formed by these variables is denoted θ^i. This problem can be viewed as problem (17) with f_i, X_i and Y_i defined as follows:

$$\begin{aligned} f_i(\theta^i) = C_2 g_i + \sum_{t=1}^{T} \Big\{ & C_1(x_i(t)+y_i(t)) + \zeta_i(t) - w_i(t) \\ & + C_5 \sum_{j\in N(i)} a_{ji}(t) - C_6 \sum_{j\in N(i)} b_{ij}(t) + C_8 \sum_{j\in N(i)} c_{ji}(t) - C_9 \sum_{j\in N(i)} d_{ij}(t) \Big\}, \end{aligned}$$

$$X_i = \left\{ \theta^i \in \Re^{N_i} \;\middle|\; \begin{array}{l} g_i, x_i(t), y_i(t), z_i(t), w_i(t), s_i(t), l_i(t), m_i(t) \geq 0, \quad \forall t, \\ a_{ji}(t), b_{ij}(t), c_{ji}(t), d_{ij}(t) \geq 0, \quad \forall t, \forall j \in N(i), \\ C_3 z_i(t) \leq \zeta_i(t),\ C_4 z_i(t) + (C_3 - C_4) g_i \leq \zeta_i(t), \quad \forall t, \\ x_i(t) \leq \bar{x}_i,\ y_i(t) \leq \bar{y}_i,\ l_i(t) \leq \bar{l}_i,\ m_i(t) \leq \bar{m}_i,\ l_i(1) = l_{i_0},\ m_i(1) = m_{i_0}, \quad \forall t, \\ z_i(t) + \sum_{j\in N(i)} a_{ji}(t) + P_i x_i(t) + \alpha_i u_i(t) + l_i(t) \\ \quad = e_i(t) + w_i(t) + \sum_{j\in N(i)} b_{ij}(t) + l_i(t+1), \quad \forall t, \\ r_i y_i(t) + \sum_{j\in N(i)} c_{ji}(t) + Q_i x_i(t) + \beta_i v_i(t) + m_i(t) \\ \quad = h_i(t) + s_i(t) + \sum_{j\in N(i)} d_{ij}(t) + \gamma m_i(t) + m_i(t+1), \quad \forall t, \\ f_i(\theta^i) \leq f_i^* \end{array} \right\},$$

$$Y_i(\theta^{-i}) = \{\theta^i \in \Re^{N_i} \mid a_{ji}(t) = b_{ji}(t),\ c_{ji}(t) = d_{ji}(t), \quad \forall (i,j) \in A\}.$$

Notice that we denote $f_i(\theta^i)$ in place of $f_i(\theta^i, \theta^{-i})$ because the objective function does not depend on the other nodes' variables θ^{-i}. Let the vector-valued functions whose elements are the functions that define X_i and $Y_i(\theta^{-i})$ be denoted as $\phi_i : \Re^{N_i} \to \Re^{L_i}$, $\psi_i : \Re^{N_i} \to \Re^{M_i}$, and $\eta : \Re^N \to \Re^H$, where L_i, M_i and H are positive integers. Then the sets X_i and $Y_i(\theta^{-i})$ are represented as follows:

$$\begin{aligned} X_i &= \{\theta^i \in \Re^{N_i} \mid \phi_i(\theta^i) \leq 0, \psi_i(\theta^i) = 0\}, \\ Y_i(\theta^{-i}) &= \{\theta^i \in \Re^{N_i} \mid \eta(\theta^i, \theta^{-i}) = 0\}. \end{aligned}$$

Therefore, problem (17) can simply be rewritten as

$$P_i(\theta^{-i}) : \quad \begin{array}{rl} \text{minimize} & f_i(\theta^i) \\ \text{subject to} & \phi_i(\theta^i) \leq 0,\ \psi_i(\theta^i) = 0, \\ & \eta(\theta^i, \theta^{-i}) = 0. \end{array} \tag{18}$$

The Karush-Kuhn-Tucker (KKT) conditions for $P_i(\theta^{-i})$, $i = 1, \cdots, K$ can be written as

$$\left\{ \begin{array}{l} \nabla_{\theta^i} f_i(\theta^i) + \nabla_{\theta^i} \phi_i(\theta^i)^T \mu^i + \nabla_{\theta^i} \psi_i(\theta^i)^T \nu^i + \nabla_{\theta^i} \eta(\theta^i, \theta^{-i})^T \lambda^i = 0, \\ \mu^i \geq 0,\ \phi_i(\theta^i) \leq 0,\ {\mu^i}^T \phi_i(\theta^i) = 0, \quad \psi_i(\theta^i) = 0, \\ \eta(\theta^i, \theta^{-i}) = 0, \end{array} \right. \tag{19}$$

where $\mu^i \in \Re^{L_i}$, $\nu^i \in \Re^{M_i}$, and $\lambda^i \in \Re^H$ are the Lagrange multiplier vectors.

In general, there is no relation among the Lagrange multiplier vectors λ^i for the shared constraints $\eta(\theta^i, \theta^{-i}) = 0$ in problem $P_i(\theta^{-i})$. A GNE is called a normalized equilibrium [7] if the following condition about the Lagrange multiplier vectors for the shared constraints are satisfied for some positive constants κ_i, $i = 1, \cdots, K$:

$$\lambda := \kappa_1 \lambda^1 = \cdots = \kappa_K \lambda^K. \tag{20}$$

In particular, if $\kappa_i = 1$, $i = 1, \cdots, K$, then (20) means that all Lagrange multipliers λ^i, $i = 1, \ldots, K$ for the shared constraints are equal.

On the other hand, the weighted total cost minimization problem can be represented as follows:

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{i=1}^{K} \omega_i f_i(\theta^i) \\ \text{subject to} & \phi_i(\theta^i) \leq 0, \ \psi_i(\theta^i) = 0, \quad i = 1, \cdots, K \\ & \eta(\theta) = 0, \end{array} \tag{21}$$

where $\omega_i > 0, i = 1, \cdots, K$ are weight factors.

Then, the following relation holds between the problem of finding a normalized equilibrium in GNEP (18) and the optimization problem (21).

Theorem *A normalized equilibrium of the GNEP in which each player i solves $P_i(\theta^{-i})$ is a solution of the optimization problem (21) for some weight factors $\omega_1, \ldots, \omega_K > 0$. Conversely, an optimal solution of the problem (21) is a normalized equilibrium of the GNEP in which each player i solves $P_i(\theta^{-i})$.*

Proof The KKT conditions for the GNEP in which each player i solves $P_i(\theta^{-i})$ are given by (19). In addition, if $\theta = (\theta^i)_{i=1}^K$ is a normalized equilibrium, there exist positive constants κ_i, $i = 1, \ldots, K$ satisfying (20) for Lagrange multiplier vectors λ^i, $i = 1, \ldots, K$ which satisfy the KKT conditions (19). Thus, by (19) and (20), we have

$$\left\{ \begin{array}{l} \nabla_{\theta^i} f_i(\theta^i) + \nabla_{\theta^i} \phi_i(\theta^i)^T \mu^i + \nabla_{\theta^i} \psi_i(\theta^i)^T \nu^i + \nabla_{\theta^i} \eta(\theta^i, \theta^{-i})^T \dfrac{\lambda}{\kappa_i} = 0, \quad i = 1, \cdots, K, \\ \mu^i \geq 0, \ \phi_i(\theta^i) \leq 0, \ {\mu^i}^T \phi_i(\theta^i) = 0, \quad \psi_i(\theta^i) = 0, \quad i = 1, \cdots K, \\ \eta(\theta^i, \theta^{-i}) = 0. \end{array} \right. \tag{22}$$

If we set $\omega_i := \kappa_i$, $\xi^i := \kappa_i \mu^i$, and $\pi^i := \kappa_i \nu^i$, then (22) can be rewritten as

$$\left\{ \begin{array}{l} \omega_i \nabla_{\theta^i} f_i(\theta^i) + \nabla_{\theta^i} \phi_i(\theta^i)^T \xi^i + \nabla_{\theta^i} \psi_i(\theta^i)^T \pi^i + \nabla_{\theta^i} \eta(\theta)^T \lambda = 0, \quad i = 1, \cdots, K \\ \xi^i \geq 0, \ \phi_i(\theta^i) \leq 0, \ {\xi^i}^T \phi_i(\theta^i) = 0, \quad \psi_i(\theta^i) = 0, \quad i = 1, \cdots, K \\ \eta(\theta) = 0. \end{array} \right. \tag{23}$$

These conditions are nothing but the KKT conditions for the optimization problem (21). Since the problem (21) is a convex programming problem, the vector θ satisfying the KKT conditions (22) is an optimal solution of (21).

Conversely, suppose that $\theta = (\theta^1, \cdots, \theta^K)$ is an optimal solution of the problem (21). Then the vector θ satisfies the KKT conditions (23) with Lagrange multiplier vectors ξ^i, π^i, and λ. By dividing the first equation of (23) by ω_i, and by setting $\frac{\xi^i}{\omega_i} =: \mu^i$, $\frac{\pi^i}{\omega_i} =: \nu^i$, and $\omega_i =: \kappa_i$, we get the KKT conditions (22) for the normalized equilibrium of the GNEP. ■

From this theorem, a solution of the weighted total cost minimization problem is a normalized equilibrium of the GNEP. In the optimization problem, each Lagrange multiplier represents the worth of the constraint associated with it. Thus, a normalized equilibrium is a special GNE such that the worth ratio of the shared constraints is identical for all nodes. This means that the ratio of relative importance of transmitting electricity and heat is equal for all nodes. If we consider the general case where the ratio is not necessarily equal among nodes, it is important to deal with GNEs other than a normalized equilibrium. For this reason, the concept of a restricted generalized Nash equilibrium (restricted GNE), which is regarded as an extension of a normalize equilibrium, has been proposed [4].

A restricted GNE is defined as a GNE in which Lagrange multiplier vectors λ^i associated with the shared constraints satisfy the condition

$$(\lambda^i)_{i=1}^K \in \Lambda,$$

where Λ is a nonempty cone in $\Re_+^{KH} := \{(\lambda^i)_{i=1}^K \in \Re^{KH} \mid \lambda^i \geq 0, i = 1, \dots, K\}$. For example, the GNEs that satisfy the following conditions can be represented as restricted GNEs:

$$\sigma_1 \kappa_i \lambda^i \leq \kappa_{i'} \lambda^{i'} \leq \sigma_2 \kappa_i \lambda^i, \quad 1 \leq i < i' \leq K, \tag{24}$$

where $\kappa_i, i = 1, \cdots, K$ are positive constants, and σ_1 and σ_2 are positive numbers such that $\sigma_1 \leq 1 \leq \sigma_2$ for $i = 1, \dots, K$. The conditions (24) mean that the ratio of the relative importance of trading electricity and heat with other nodes lies in a certain range for each pair of nodes. In particular, if $\sigma_1 = \sigma_2 = 1$, then $\kappa_1 \lambda^1 = \kappa_2 \lambda^2 = \cdots = \kappa_K \lambda^K$. Hence, the restricted GNE is a special case of a normalized equilibrium. The method for computing a restricted GNE is proposed in [4].

4 Numerical experiments

In this section, we present some numerical results to examine the effect of energy transmission in the CHP network. The experiments are conducted for the following three models. In the first model, which is referred to as Model 0, no energy is assumed to be transmitted among nodes and each node minimizes its own cost by itself. By solving Model 0, we get the standard values $f_i^*, i = 1, \dots, K$ used in the constraint (15). The second is the total cost minimization model with energy transmission as formulated in Section 2, which we call Model 1. Finally, the third model is the same as Model 1 except that the constraint (15) is removed, which is called Model 2.

We set the planning horizon as $t = 1, \cdots, 24$, the node set as $N = \{1, 2, 3\}$, and the arc set as $A = \{(1, 2), (1, 3), (2, 3)\}$. We use the values shown in Table 1 as the constants in the total cost minimization problem. We explain how to set the unit price of electricity and heat transmitted among nodes. We multiply the unit price C_5 of electricity purchased from the electric power company multiplied by 0.95 to set the unit price C_6 of electricity sold to other nodes, and multiply C_6 by 0.95 to determine the unit price C_7 of electricity sold to the electric power company. As for the price of heat transmitted among nodes, we multiply the price of heat produced by a gas boiler by 0.95 to determine the unit price C_7 of heat purchased from other nodes, and multiply C_7 by 0.95 to get the unit price C_8 of heat sold to other nodes.

Table 1: Data used in the experiment

constant	value	unit	constant	value	unit
C_1	9.400×10^{-3}	yen/Wh	$\bar{x}_i$	4,926.1	Wh
C_2	8.700×10^{-3}	yen/Wh·day	$\bar{y}_i$	36,000	Wh
C_3	2.000×10^{-2}	yen/Wh	$\bar{l}_i$	16,000	Wh
C_4	4.000×10^{-2}	yen/Wh	$\bar{m}_i$	7,222	Wh
C_5	1.900×10^{-2}	yen/Wh	α_i	20	m^2
C_6	1.805×10^{-2}	yen/Wh	β_i	5	m^2
C_7	1.715×10^{-2}	yen/Wh	γ	0.02	—
C_8	9.400×10^{-3}	yen/Wh	P_i	0.203	—
C_9	8.900×10^{-3}	yen/Wh	Q_i	0.569	—
			R_i	0.95	—

The energy production data of a solar battery and a solar heat panel, which are common to all nodes, are shown in Fig.4 and Fig.5, respectively. The data of the electricity demand and the heat demand are shown in Fig.6 and Fig.7, respectively. This demand pattern for electricity and heat is called pattern I. We perform the experiments for Model 0, Model 1, and Model 2 under these conditions. The results of the experiments such as the input quantities of a gas engine and a gas boiler, the purchase amount of electricity and gas from the electricity and gas companies, the sales amount of electricity to the electricity company, the amount of electricity and heat storage in a battery and a tank, and the quantity of energy transmission among nodes, are described in the graphs in the Appendix. We solved the linear programming problem by the ILOG CPLEX® solver.

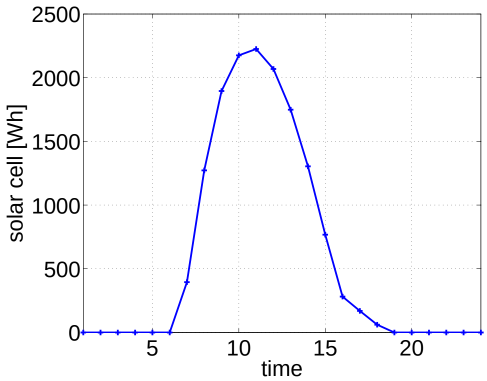

Fig. 4: Electricity production of a solar battery

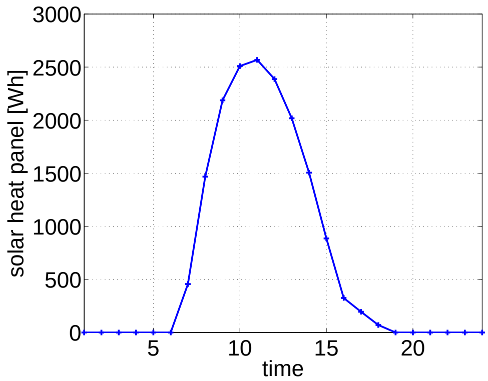

Fig. 5: Heat production of a solar heat panel

The minimum cost of each node and total cost of the network are summarized in Table 2. The numbers in parentheses represent the percentage reduction from the cost of Model 0. Table 2 shows that the cost of Model 1 with energy transmission is, of course, not greater than the cost of Model 0 with no transmission. That is due to the constraint (15) which requires the cost f_i not to exceed f_i^* at each node. In terms of each energy cost, there is no change in the cost of node 1, but the costs of node 2 and node 3 have been reduced in Model 1 because their surplus

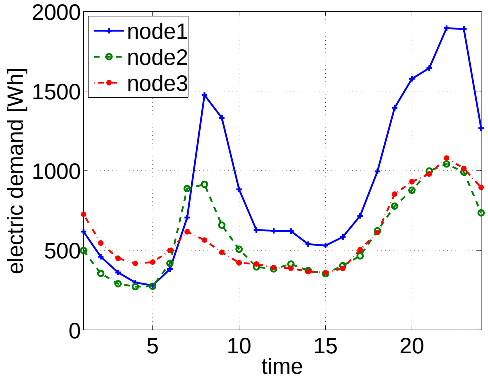

Fig. 6: Electricity demand (pattern I)

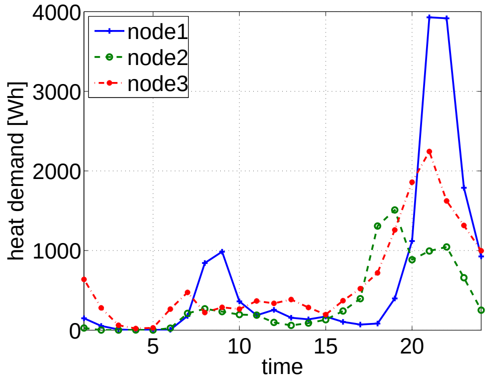

Fig. 7: Heat demand (pattern I)

electricity, which was sold to the electricity company in Model 0, can be sold to node 1 through the energy transmission.

In Model 2, the total cost is also not greater than that of Model 1. However, the cost of node 1 is greater in Model 2 than in Model 1. We can see that the cost of node 1 is sacrificed in Model 2 because the total benefit is prioritized. Therefore, by imposing the constraint (15) of Model 1, we can obtain a solution such that the cost of each node remains within the standard value of Model 0, and the total cost is almost the same as that of Model 2.

Table 2: Comparison of costs in models (pattern I)

	f_1	f_2	f_3	Total
Model 0	267.89	57.48	126.09	451.456
Model 1	267.89 (0.00%)	54.48 (-5.23%)	124.66 (-1.13%)	447.028 (-0.9808%)
Model 2	268.09 (0.08%)	54.43 (-5.31%)	124.51 (-1.26%)	447.026 (-0.9813%)

Next, we change the electricity demand and the heat demand as shown in Fig.8 and Fig.9, respectively. We refer to these demands as pattern II. Note that the electricity and heat demands of nodes 2 and 3 are the same as those in pattern I. As to the electricity and heat demands of node 1, the total amount is the same in both patterns, although the demand pattern is different. This means that there is a node which has a quite different nature. In Table 3, we show the numerical results for Model 0 and Model 1 with demand pattern II.

Table 3: Comparison of costs between Model 0 and Model 1 (pattern II)

	f_1	f_2	f_3	Total
Model 0	264.41	57.48	126.09	447.98
Model 1	263.05 (-0.51%)	54.46 (-5.24%)	123.37 (-2.16%)	440.88 (-1.58%)

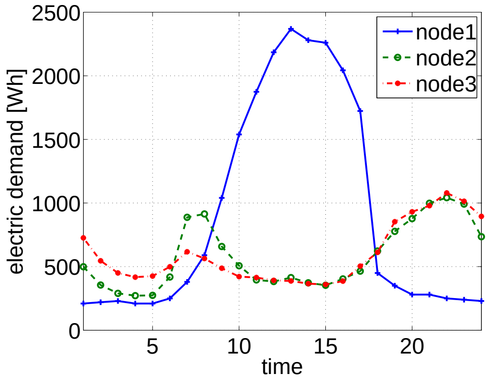

Fig. 8: Electricity demand (pattern II)

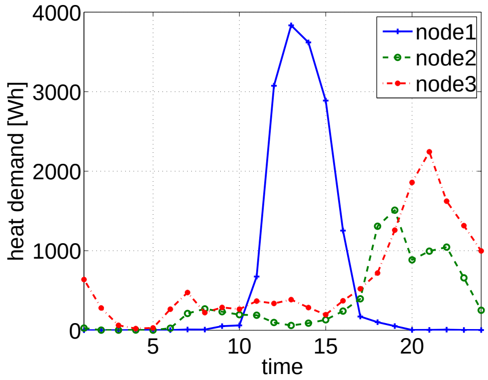

Fig. 9: Heat demand (pattern II)

Table 3 shows that the cost of each node and the total cost of Model 1 are both less than those of Model 0. The amount of electricity transmission and heat transmission to other nodes are shown in Fig.10 and Fig.11, respectively. The total quantities of electricity purchase and electricity sales with the electricity company for Model 0 and Model 1 are shown in Fig.12 and Fig.13, respectively. From these figures, we can see that in Model 1, the total cost of all nodes is less compared with Model 0, since electricity and heat are effectively transmitted among nodes and the amount of electricity traded with the electricity company is reduced. Notice that the strict inequality in the constraint (15) holds for each node in the solution of Model 1. This fact means that, for demand pattern II, the solution of Model 1 is the same as that of Model 2 which does not have the constraint (15).

Let us compare the effect of the energy transmission among nodes for the two demand patterns. The percentage of total cost loss in Model 1 compared with Model 0 is 0.9808% for demand pattern I, and is 1.5829% for demand pattern II. Energy sales among nodes with pattern I and pattern II are shown in Fig.14 – Fig.19. From these figures, we can see that the amount of energy transmission for pattern II is greater than that for pattern I. Hence, the existence of a node whose demand is quite different from other nodes in the CHP network can decrease the total energy cost by effective energy transmission.

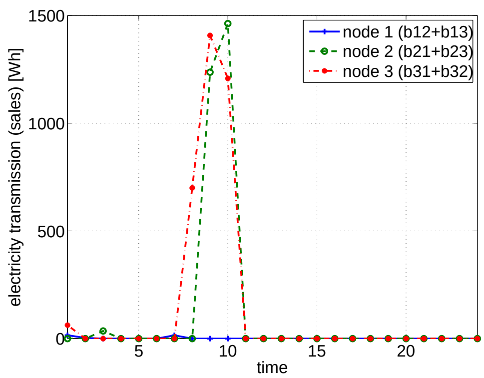

Fig. 10: Electricity transmission to other nodes

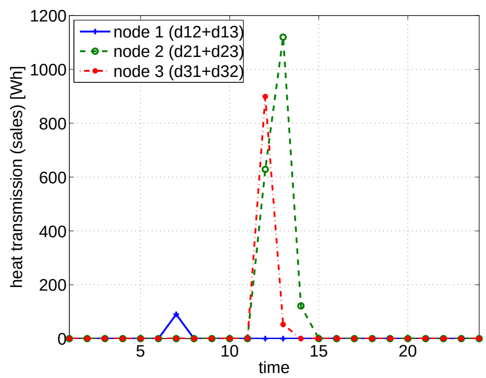

Fig. 11: Heat transmission to other nodes

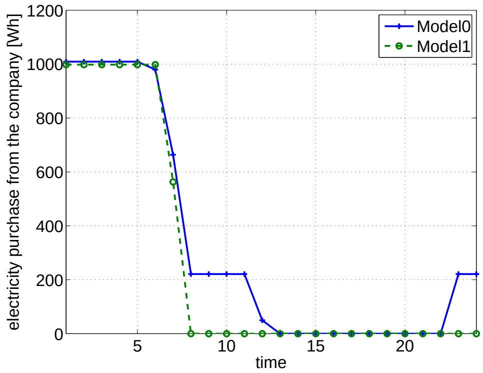

Fig. 12: Electricity purchase from the company

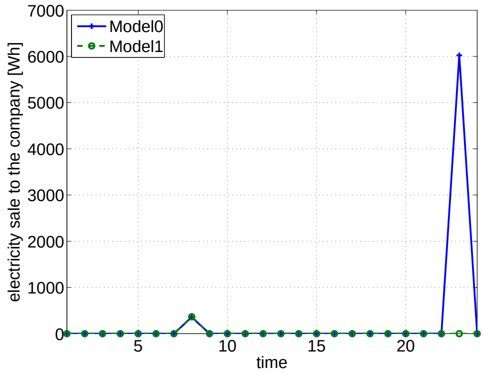

Fig. 13: Electricity sales to the company

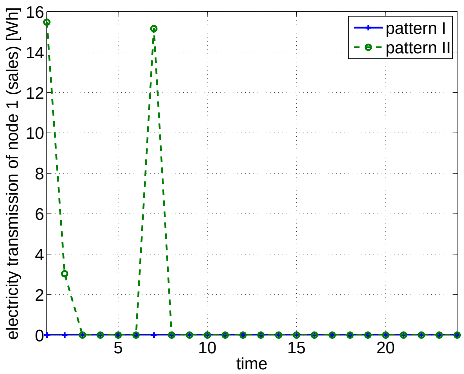

Fig. 14: Electricity transmission of node 1 (sales)

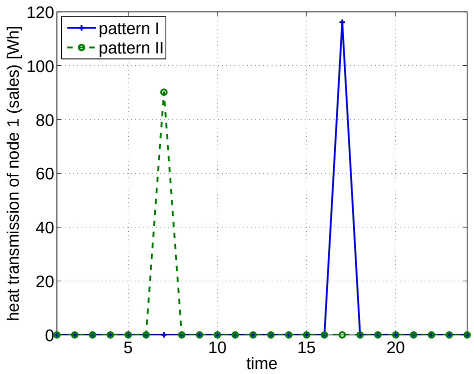

Fig. 15: Heat transmission of node 1 (sales)

5 Concluding remarks

We have formulated the problem in which the total cost of the CHP network is minimized. Moreover, from the game theoretic viewpoint, we have also formulated the problem as a GNEP in which each node minimizes its own cost. We have proved that an optimal solution of the weighted total cost minimization problem is a normalized equilibrium of the GNEP. We have confirmed the cost reduction effect of energy transmission by numerical experiments.

It is an interesting subject of future research to formulate the problem as a stochastic programming problem under the condition that some factors, such as the energy demand, the electric production of a solar battery, and the heat production of a solar heat panel, are subject to uncertainty.

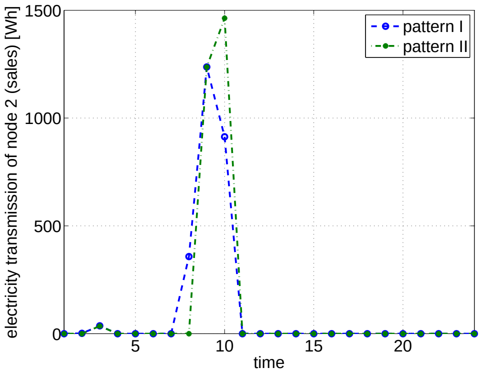

Fig. 16: Electric transmission of node 2 (sales)

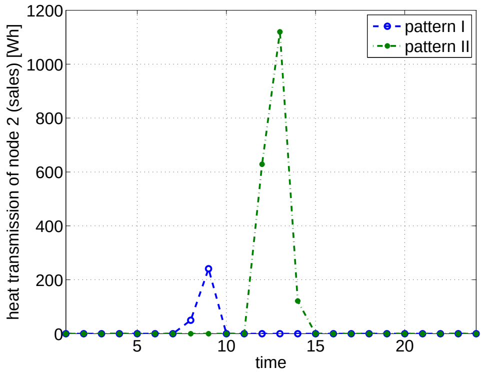

Fig. 17: Heat transmission of node 2 (sales)

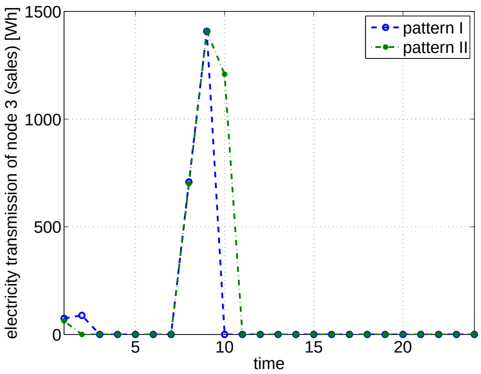

Fig. 18: Electric transmission of node 3 (sales)

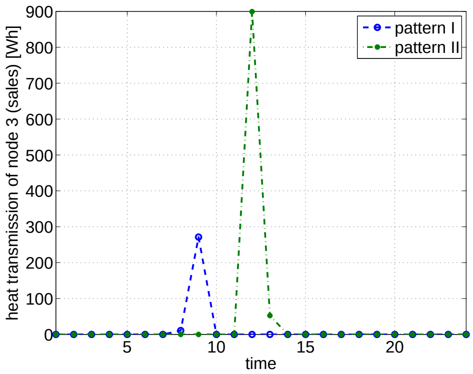

Fig. 19: Heat transmission of node 3 (sales)

Acknowledgment

I would like to express my sincere appreciation to Professor Masao Fukushima, who supervised me throughout my bachelor's and master's course. He kindly supported me and gave me valuable comments on my research. Without his precise guidance and constructive suggestions, this paper would not have been possible. I would also like to thank Dr. Yoichi Tanaka of TOHO GAS Co., Ltd. He gave me a chance to try this subject and helpful comments for my research. I am thankful to Associate Professor Nobuo Yamashita. He gave me helpful comments for my research. I also wish to express my thanks to Assistant Professor Shunsuke Hayashi. He gave me appropriate comments for my study. Moreover, I would like to express my gratitude to all the members in Fukushima Laboratory. Finally, I would like to thank my parents for their great support.

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A Detailed numerical results

We show the results of numerical experiments below. We omit the results for waste heat because it is constantly equal to zero at all nodes.

A.1 Results for Model 0 (pattern I)

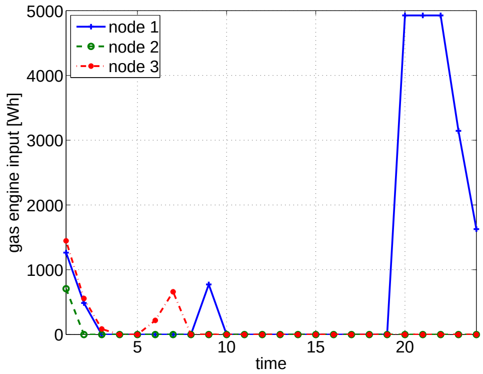

Fig. 20: Input of a gas engine

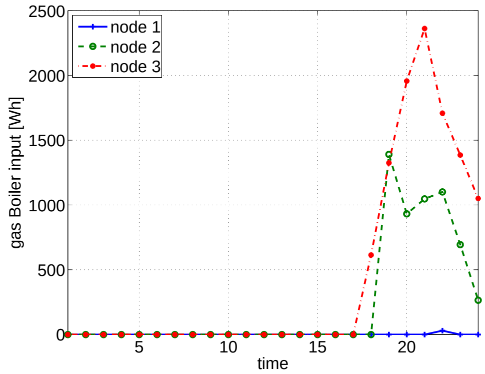

Fig. 21: Input of a gas boiler

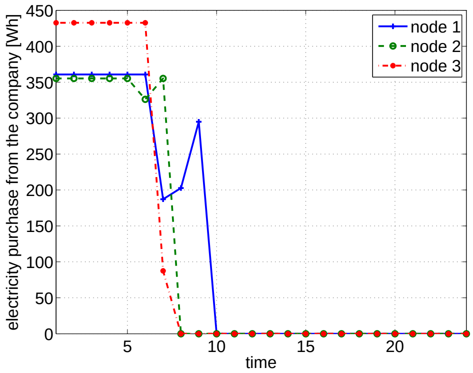

Fig. 22: Electricity purchased from the electricity company

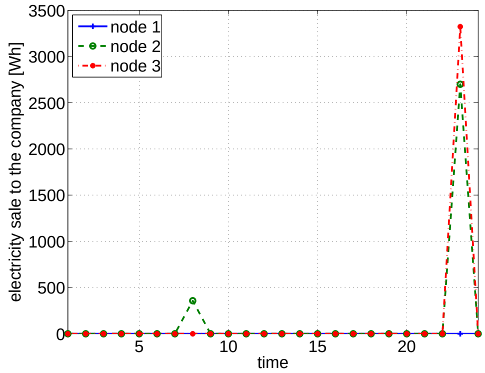

Fig. 23: Electricity sold to the electricity company

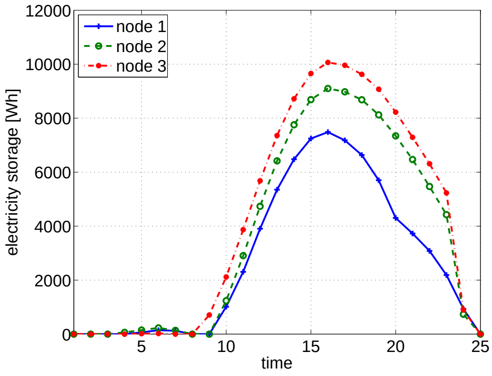

Fig. 24: Storage volume of electricity

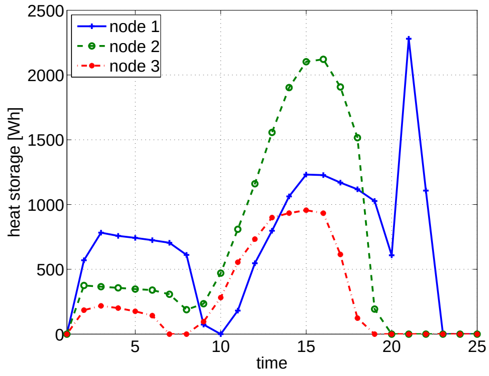

Fig. 25: Storage volume of heat

A.2 Results for Model 1 (pattern I)

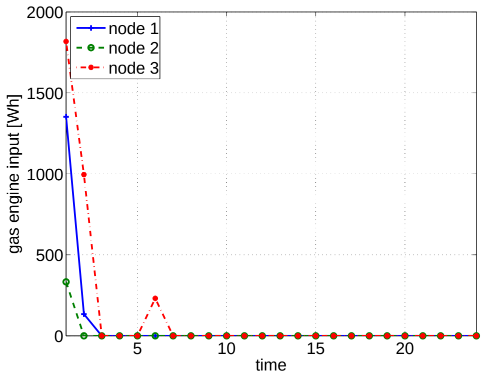

Fig. 26: Input of a gas engine

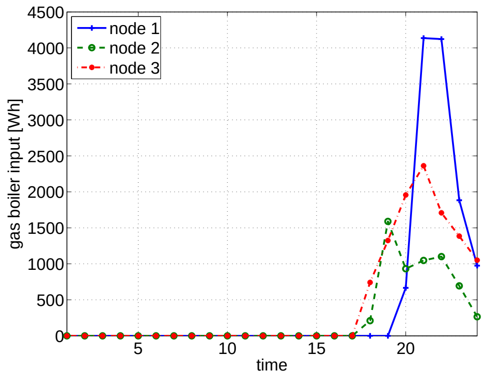

Fig. 27: Input of a gas boiler

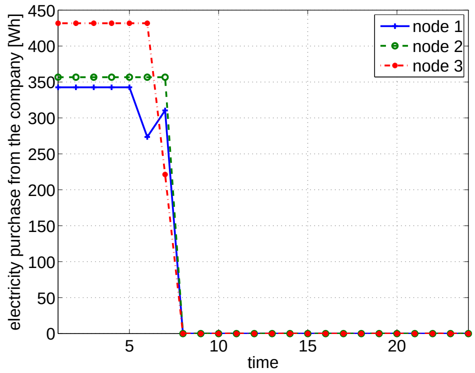

Fig. 28: Electricity purchased from the electricity company

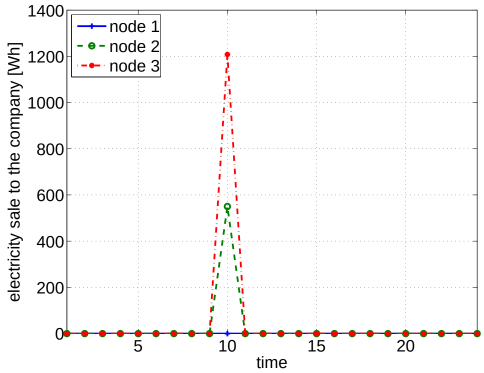

Fig. 29: Electricity sold to the electricity company

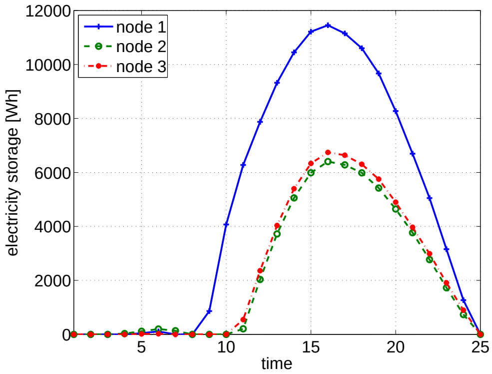

Fig. 30: storage volume of electricity

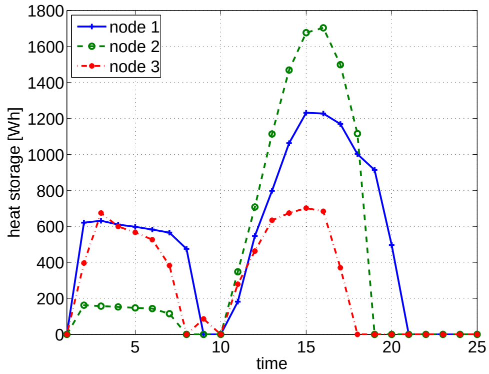

Fig. 31: Storage volume of heat

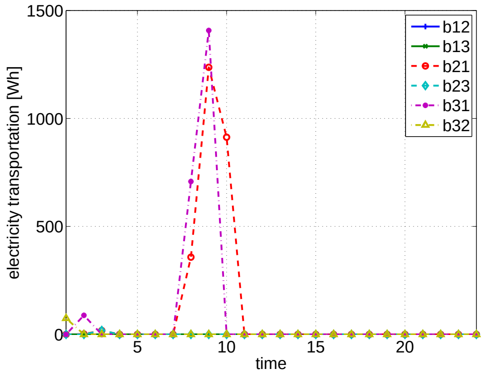

Fig. 32: Electricity transmission among nodes

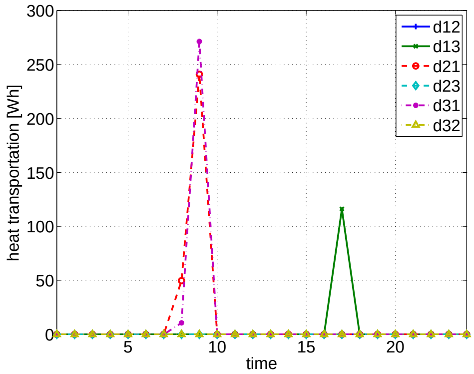

Fig. 33: Heat transmission among nodes

A.3 Results for Model 2 (pattern I)

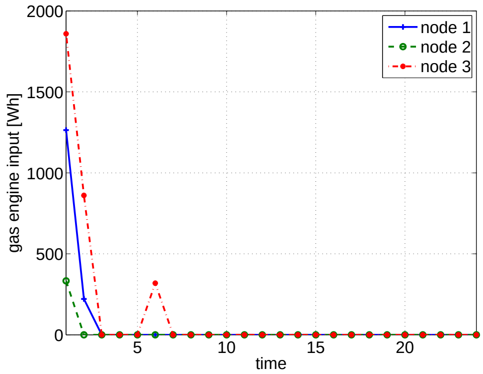

Fig. 34: Input of a gas engine

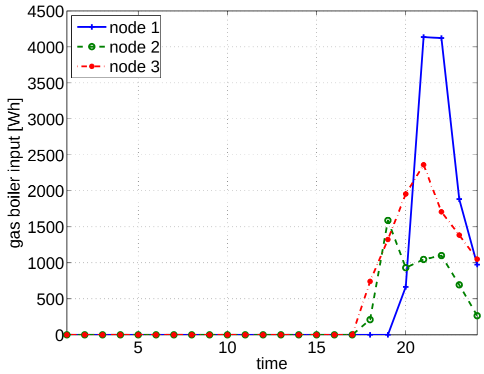

Fig. 35: Input of a gas boiler

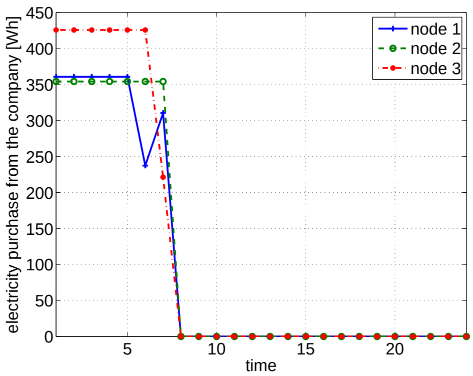

Fig. 36: Electricity purchased from the electricity company

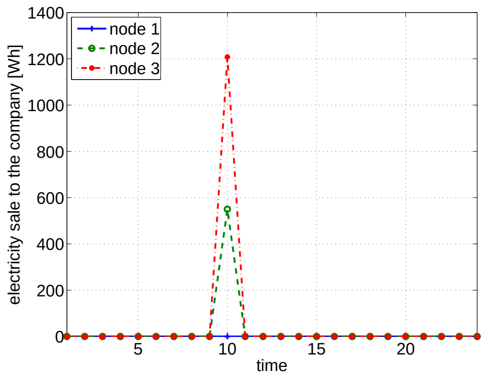

Fig. 37: Electricity sold to the electricity company

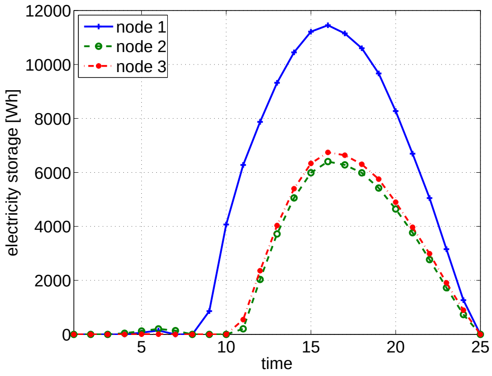

Fig. 38: Storage volume of electricity

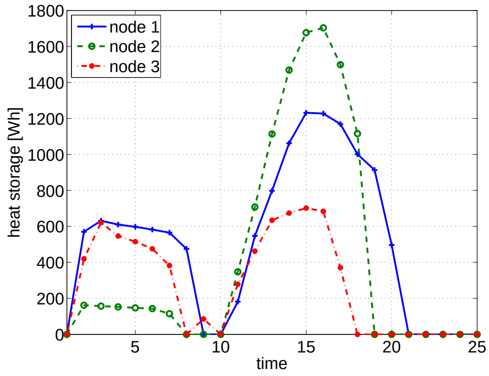

Fig. 39: Storage volume of heat

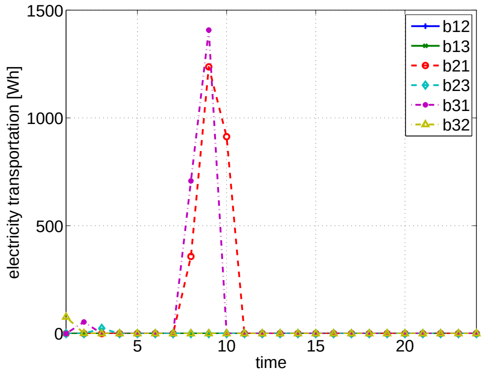

Fig. 40: Electricity transmission among nodes

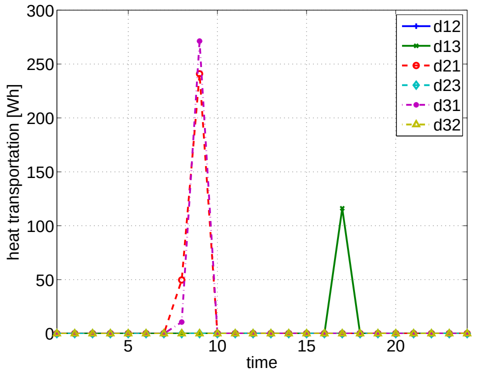

Fig. 41: Heat transmission among nodes

A.4 Results for Model 0 (pattern II)

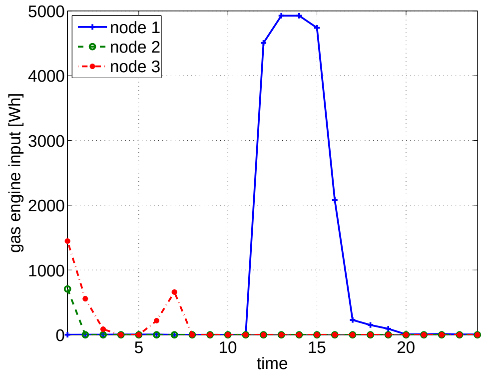

Fig. 42: Input of a gas engine

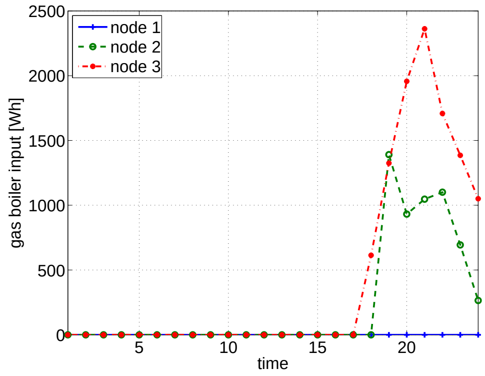

Fig. 43: Input of a gas boiler

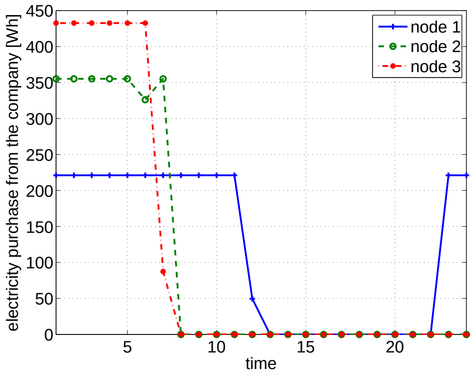

Fig. 44: Electricity purchased from the electricity company

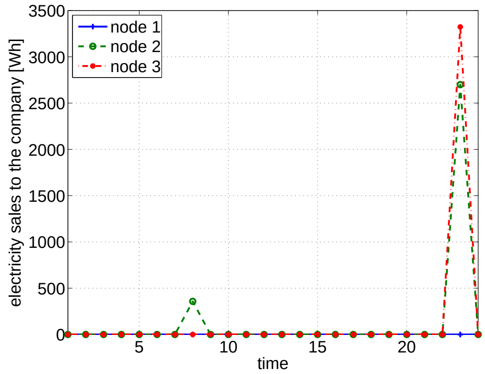

Fig. 45: Electricity sold to the electricity company

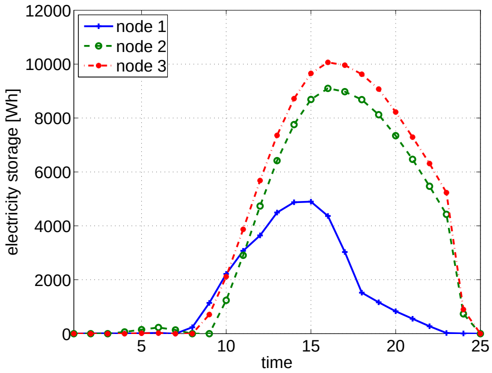

Fig. 46: Storage volume of electricity

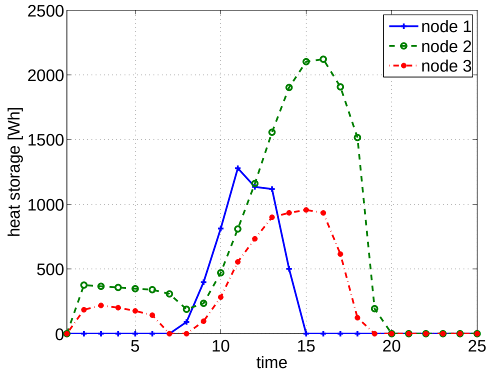

Fig. 47: Storage volume of heat

A.5 Results for Model 1 (pattern II)

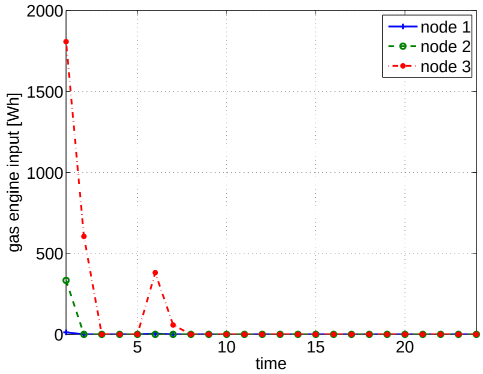

Fig. 48: Input of a gas engine

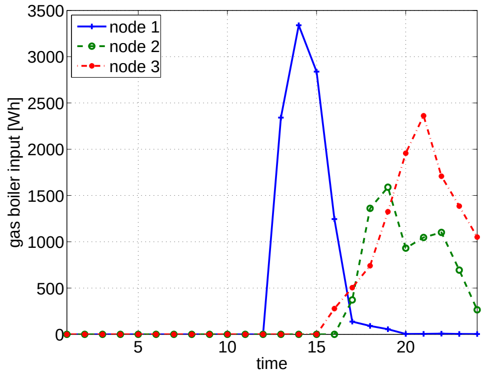

Fig. 49: Input of a gas boiler

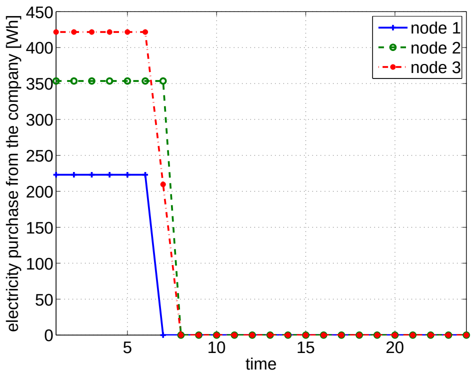

Fig. 50: Electricity purchased from the electricity company

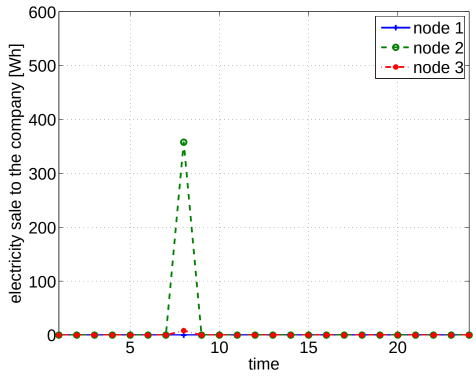

Fig. 51: Electricity sold to the electricity company

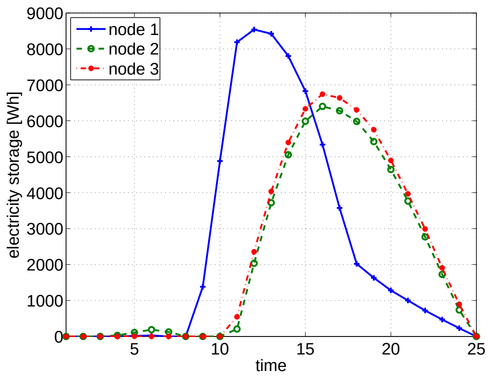

Fig. 52: Storage volume of electricity

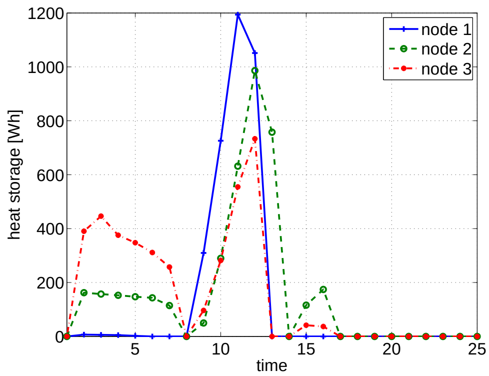

Fig. 53: Storage volume of heat

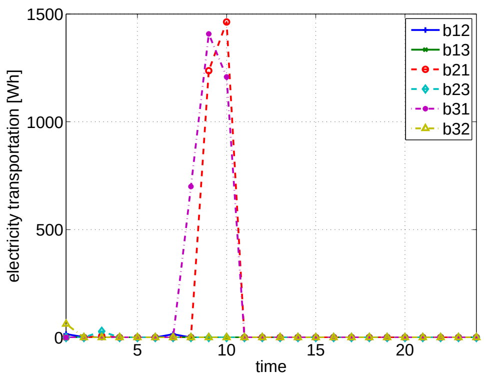

Fig. 54: Electricity transmission among nodes

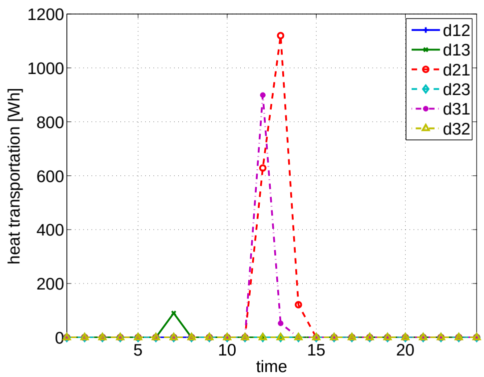

Fig. 55: Heat transmission among nodes