Master's Thesis

Efficient Block Selections in Block Coordinate Gradient Descent Methods for Linearly Constrained Optimization Problem

Guidance

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Abstract

A block coordinate gradient descent method (BCGD) is frequently used for the large-scale optimization problem. It updates a block (set of variables) in each iteration with other variables fixed.

In this paper we consider BCGD for optimization problem with linearly equality constraints. When the block is not selected properly, BGCD does not necessarily converge globally. To guarantee the global convergence, we may select a block that satisfies the Gauss-Southwell-q Rule. However, we cannot adopt the rule for the problems with more than three equality constraints, because the computational cost for finding a block satisfying the rule is $O(n^2)$ in each iteration.

In this paper, we propose novel methods of selecting blocks which incorporates both low computational cost and the global convergence. The proposed method first selects a set of the blocks before starting the BCGD. Therefore, we do not need to construct a block in each iteration. Then BCGD selects a block from the set in a cyclic, greed or random manner. We show the global convergence of the proposed method. Moreover, numerical experiments are conducted to investigate its validity. These numerical results support the global convergence of the proposed method with a little computational cost for selecting blocks.