Master's Thesis

A Forward-Backward Splitting Method with Component-wise Lazy Evaluation for Online Structured Convex Optimization

Guidance

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Abstract

 We consider large-scale optimization problems whose objective function is expressed as the sum of convex functions. Such problems appear in various fields, such as statistics and machine learning. One of the solution methods for the problems is the online gradient method, which exploits gradients of a few functions in the objective function at each iteration.

In this paper, we focus on sparsity of two vectors: a solution and a gradient of each function in the objective function. To get a sparse solution, we usually add the L1 regularization term to the objective function. Then, since the L1 regularization term consists of all decision variables, the usual online gradient methods update all variables even if each gradient is sparse. To reduce the computations of all variables at each iteration, we may use a lazy evaluation, which only updates the variables corresponding to the nonzero components of the gradient by ignoring the L1 regularization terms, and evaluates the ignored ones later. Since the lazy evaluation does not exploit the information of the terms at all iterations, the online algorithm with the lazy evaluation might converge slowly.

In this paper, we propose a forward-backward splitting type method that exploits the L1 terms corresponding to the nonzero components of the gradient at each iteration. We show that its regret bound is $O(\sqrt{T})$, where T is the number of functions in the objective function. Moreover, we report numerical experiments for the proposed method, which show the proposed method is promising.