Master's Thesis

An equivalent nonlinear optimization model with triangular low-rank factorization for semidefinite programs

Guidance

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Abstract

Semidefinite programming (SDP) problems have a lot of applications such as LMI-constrained problems and minimal eigenvalue problems and thus, have been studied extensively. One of the most popular method for solving SDP is the interior-point method, where a solution can be obtained in polynomial time. However, it is not applicable to large-scale problems, and it may not get high accurate solutions in certain SDPs. To solve large-scale problems with moderate accuracy, Burer and Monteiro reformulated an SDP problem with a nonlinear optimization model using fewer decision variables, and proposed an augmented Lagrangian method for solving it. However, since the local optimal solution of the reformulated problem is always non-unique, it is difficult to construct solution methods that theoretically guarantee fast convergence.

In this paper, we consider another reformulation of SDP problems, using a new nonlinear optimization model with the decision variable being a triangular low-rank matrix. In this way, we can further reduce the number of variables compared to the one proposed by Burer and Monteiro. In addition, we can guarantee local uniqueness of the optimal solution under certain conditions. In order to obtain a highly accurate solution, we consider the sequential quadratic programming method for both nonlinear optimization models, and discuss ways to make the implementation efficient. Finally, we present some numerical results, which show that the proposed method is promising.