Master's Thesis

An active set method for $\ell 1$-regularized optimization

Guidance

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Abstract

In recent years, machine learning and compressed sensing technologies have been used in various fields. The $\ell 1$-regularized problem is one of the key optimization problems in such technologies. This problem has two purposes: minimizing some loss function, which represents an error between the prediction model and the data, and obtaining a sparse parameter of the prediction model. In this paper, we consider an $\ell 1$-regularized problem that has an upper bound constraint on the value of loss functions, since it enables us to control the error in advance. For this problem, there are many methods. However, they may not obtain highly accurate solutions when the problem is large-scaled.

In this paper, we propose an active set method for the Fenchel dual of the $\ell 1$-regularized problem. Moreover, we show that the proposed method finds an exact solution with finite iterations, when the subproblems of the proposed method are solved exactly. Thus, we can expect the proposed method to successfully obtain an optimal solution with high accuracy.

Furthermore, we consider a particular case of $\ell 1$-regularized problem called Basis Pursuit Denoising (BPDN), and give an efficient and concrete implementation of the active set method for it. By numerical comparison with an existing method called SPGL1, we show that the proposed method can solve the problem faster than it, when the optimal solution is strongly sparse. We also obtain a solution for BPDN with higher accuracy.