Master's Thesis

Differentiable Exact Augmented Lagrangian Functions for Nonlinear Second-Order Cone Programs

Guidance

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Abstract

The nonlinear second-order cone programming (SOCP) generalizes the well-known nonlinear programming by considering second-order conic constraints. It has applications in many fields, including engineering and finance. In particular, the linear SOCP, where the objective and constraint functions are all linear, has been studied considerably. The research about nonlinear SOCP is more recent, and methods that reformulate the original problem into one or a sequence of unconstrained problems, using a penalty function, have been also proposed. One of them is the differentiable exact penalty method proposed in 2012. In this case, the term “exact” means that an optimal solution can be obtained by solving the reformulated problem only once, if a certain parameter is chosen appropriately.

In this work, we propose a differentiable exact augmented Lagrangian function for nonlinear SOCP. The difference between the exact penalty function and the exact augmented Lagrangian is that the former has the same number of variables as the original problem, and the latter additionally considers the Lagrange multipliers as variables. In the case of exact penalties, a function that estimates the Lagrange multipliers is incorporated in their formula. So, their evaluation require to solve linear least squares problems, and thus, to do matrix factorizations. On the other hand, for the proposed exact augmented Lagrangian function, the multiplier estimate is used only to establish the idea of its construction and to facilitate the exactness proof. This means that no matrix factorizations are required for the evaluation of our proposed function. In this paper, we propose an exact augmented Lagrangian function for SOCP and prove its exactness under the so-called nondegeneracy condition. In addition, we show results of some numerical experiments solving the unconstrained reformulations with a quasi-Newton method.