Master's Thesis

An Efficient Difference of Convex Functions Algorithm for Nonlinear Conic Programming

Guidance

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February 2019

Abstract

Nonlinear conic programming (NCP) generalizes nonlinear programming (NLP), nonlinear second-order cone programming (NSOCP) and nonlinear semidefinite programming (NSDP). These problems have applications in many areas, including robust optimization, combinational optimization, control theory and eigenvalue problems. In this paper, we focus on nonconvex NCP problems. One approach for solving nonconvex optimization problems is the difference of convex functions algorithm (DCA), which has been developed for NLP and NSDP. In particular, Dinh *et al.* proposed recently a DCA for NSDP problems with bilinear matrix inequality constraints. However, no DCA was proposed for NCP. DCAs obtain local optimal solutions by solving convex subproblems iteratively. The subproblem consists in replacing the nonconvex function with its difference of convex (DC) decomposition, then the concave term of this decomposition is linearized. Moreover, a regularization term is added to the original objective function. However, the DC decomposition is fixed during all iterations.

In this paper, we propose a new efficient DCA that allows us to use different DC decompositions in each iteration. Such an idea is new for the DCAs for NLP and NSDP, but here we also consider the new DCA for the more general NCP problems. In this work, we first show that many classes of functions have in fact DC decompositions. In particular, we show that twice continuously differentiable vector-valued and matrix-valued functions have DC decompositions with respect to the ordering defined by the second-order and semidefinite cones, respectively. After that, we prove the global convergence of the new DCA under a weak condition. More precisely, instead of fixing the regularization parameters, as it was required in the convergence results of standard DCAs, we allow them to vary in each iteration. Therefore, we establish the convergence assuming either that the parameters are fixed or considering that they converge to zero. We conclude from the numerical experiments that the new DCA is promising, since it obtains better solutions than the ones obtained by the DCA with fixed DC decompositions and regularization parameters.