

Master's Thesis

Positively Homogeneous Optimization on Inner Product  
Spaces and its Duality

Guidance

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## Abstract

Recently, Yamanaka and Yamashita considered the positively homogeneous optimization (PHO) on  $\mathbb{R}^n$ , which includes the absolute value optimization, and proposed its dual problem. They proved that the weak duality holds under some mild conditions and gave sufficient conditions under which the proposed dual and the Lagrange dual problems are equivalent. Moreover, they showed that the strong duality and other interesting properties hold for the gauge optimization problem which is a convex PHO.

In this work, we extend PHO with inequality constraints on  $\mathbb{R}^n$  to the one on a generalized inner space with conic constraints. Note that the conic constraints are generalization of inequality constraints. Then the extended problem includes the semidefinite programming and the nuclear norm minimization. We propose its dual problem and show the weak duality and the equivalence to the Lagrange dual problem. Moreover, we prove that the strong duality holds when the extended problem is convex. Finally, we discuss the sufficient and necessary optimality conditions associated to the problem.