

Master's Thesis

Nonmonotone Descent Methods for Multiobjective  
Optimization Problems

Guidance

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February 2019

## Abstract

Multiobjective optimization problems are optimization problems with several objective functions. In the last two decades, many descent methods, such as the steepest descent, the Newton, and the projected gradient methods, were proposed for such problems. These methods compute the search directions by solving certain convex subproblems. Then they choose the stepsizes by using the Armijo-like line search, so that all the objective functions values decrease in each iteration.

In this paper, we consider nonmonotone line searches for constrained multiobjective optimization, which allow some increase in the objective functions values. More precisely, we focus on two nonmonotone schemes: the one that uses the maximum of the recent functions values and the one that uses the weighted average of all the past functions values. Moreover, we propose a new nonmonotone line search specifically for multiobjective problems, which can be viewed as a combination of monotone and nonmonotone techniques. We prove that a sequence generated by these nonmonotone approaches converges globally to a Pareto critical point, when the search directions satisfy some specific but general conditions. We show that these conditions hold for the steepest descent, the Newton, and the projected gradient directions. Although we establish the convergence for general directions, we observe in particular that this is the first globally convergent result for the projected gradient method with the maximum-type nonmonotone line search. Finally, we confirm the efficiency of the nonmonotone technique for multiobjective optimization with some numerical experiments.