Master's Thesis

A Stochastic Projection Method with Variance Reduction for Variational Inequality Problems

Guidance

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Abstract

The variational inequality problem (VIP) is one type of equilibrium problems, and has applications in various fields such as economics, game theory, and engineering. In this paper, we focus on VIP whose mapping F is defined as a sum of a large number of mappings F_i. Recently, such VIPs are used for formulating the problems from data analysis and machine learning fields. A popular approach for solving VIPs is the stochastic projection method. It is an iterative method that generates the next iterate using only a single mapping F_i chosen randomly. Since the expectation of F_i is the full mapping F, the method converges globally if a diminishing stepsize rule is adopted. However, the convergence is slow because the variance of F_i is not zero in general, and thus the stepsize should approach to zero.

In this paper, we propose a stochastic projection method with variance reduction, using the idea of stochastic variance reduced method for convex optimization. We show its global convergence under mild conditions on the stepsizes. Moreover, we propose novel stepsize rules that satisfy these conditions and give a larger stepsize.

We report some numerical experiments for the proposed method with these stepsize rules. In paricular, we solved a stochastic VIP derived from the Karush-Kuhn-Tucker conditions of a support vector machine problem. Since the VIP is box-constrained, the proposed method can be easily implemented. Numerical results show that the proposed projection method with variance reduction is faster than the existing one. In particular, we observe that stepsizes calculated by the proposed stepsize rules become larger and that they make the method more efficient.