

Master's Thesis

# Sparse solutions of Affine Variational Inequality

Guidance

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## Abstract

In this paper, we consider sparse solutions of affine variational inequality (AVI). The sparse solutions of AVI are useful for constructing a mixed strategy equilibrium in non-cooperative games and a nonlinear decoding in signal processing. When a matrix involved in AVI is positive semidefinite, a solution set of the AVI is represented as a polyhedron with using a single solution of the AVI. Then we can formulate an optimization problem to find a sparse solution as a minimization of a number of nonzero elements over the solution set. Unfortunately the problem is NP-hard. Thus we propose a convex relaxation problem whose objective function is the  $l_1$  norm of variables instead of the number of the nonzero elements. Concretely we propose a 2-step algorithm: the first step finds a solution of AVI, and then the second step finds a sparse solution by minimizing the  $l_1$  norm over the solution set formulated with the solution obtained in the first step. This approach is a natural extension of Chen and Xiang's work, where they consider the linear complementarity problem. We also discuss a theoretical property of sparse solutions of AVI. Moreover, we present some numerical results for the proposed approach.