Master's Thesis

On the Use of Differentiable Exact Penalty Functions for Nonlinear Semidefinite Programming

Guidance

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Abstract

The nonlinear semidefinite programming problem (NSDP) is an extension of nonlinear programming (NLP), nonlinear second-cone programming (NSOCP) and linear semidefinite programming. NSDP has applications in various fields, such as control theory, truss design problems and finance. Differently from the linear semidefinite programming case, there are still few methods proposed for NSDP. We can cite, for example, the interior-point method, the sequential quadratic programming, the augmented Lagrangian, and the penalty-type method. In this work, we focus on the penalty-type method, in particular, the exact penalty one. It consists in replacing the original problem with an unconstrained minimization of the so-called penalty function. By choosing an appropriate penalty parameter, the original problem can be solved by minimizing the penalty function only once.

Differentiable exact penalty-type methods have been proposed for NLP since 1980's, were extended recently to NSOCP by Fukuda, Silva and Fukushima (2012), and were further extended to NSDP by Han (2014). Our objective is to develop Han's work to make the method implementable. In particular, the proposed exact penalty function for NSDP depends on the derivatives of the objective and constraints functions. This means that Newton-type methods need to deal with third-order derivatives. Moreover, no discussions about the penalty parameters and no numerical experiments were done in Han's work. Therefore, here (a) we propose a modified Newton-type method that avoids those third-order derivatives, (b) we prove that the method converges globally with superlinear convergence rate, and (c) we show a way to update the penalty parameters. In addition, we perform some preliminary numerical experiments to check the validity of the exact penalty function, comparing also with a similar method called exact augmented Lagrangian.