Master's Thesis

Approximated Logarithmic Maps on Riemannian Manifolds with Application to Optimization Problems

Guidance

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Abstract

Recently, optimization problems on Riemannian manifolds involving geodesic distances such as Riemannian clustering and dictionary learning have been attracting considerable research interest. To compute geodesic distances and their Riemannian gradients, we can use logarithmic maps. However, the computational cost of logarithmic maps on Riemannian manifolds is generally higher than that on the Euclidean space. To overcome this computational issue, we propose an approximated logarithmic map following the definition of a retraction, which is an approximation of the Riemannian exponential map and is used in an update formula of iterative methods of Riemannian optimization. We prove that the definition is locally equivalent to an inverse retraction. This enables to use approximated logarithmic maps as inverse retractions, and contributes to algorithms like the conjugate gradient (CG) method with inverse retraction, which requires an inverse retraction, not the retraction itself.

Numerical experiments of computing the Riemannian center of mass with approximating the gradient of the objective function show that the proposed approximation significantly reduces the computational time to perform the gradient descent (GD) method while maintaining appropriate precision if the data diameter is sufficiently small. In addition, especially on the manifold of symmetric positive definite matrices (SPD manifold), we prove that approximation for the gradient in the GD algorithm gives a descent direction in the first iteration. Furthermore, numerical experiments focusing on CG methods show that the CG method with an approximated logarithmic map accomplishes faster convergence than the traditional CG with a vector transport on the SPD manifold.