Master's Thesis

Riemannian Generalized Newton Methods with Retractions for Nonsmooth Equations

Guidance

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Abstract

The Newton method is known as a quadratic convergent algorithm for unconstrained optimization problems in Euclidean spaces. However, it cannot be applied to solving constrained nonlinear equations and constrained problems as it is. When constraints have geometrically favorable properties, we can naturally apply unconstrained optimization methods to constrained optimization problems by regarding them as unconstrained optimization problems on Riemannian manifolds. This is called Riemannian optimization and has been actively studied in recent years due to its abundant application examples. The Newton method has been extended to that on Riemannian manifolds, and the generalized Newton method, which is applicable to wider functions than the Newton method, is also expected to be extended to that on Riemannian manifolds.

When the generalized Newton method is used for solving nonlinear equations, the method is applicable to semismooth functions wider than C^1 functions, which are in a wider class than the class C^1. Whereas when the method is used for solving optimization problems, the method is applicable to SC^1 functions, whose class is wider than the class C^2. Some studies have addressed the extension of the generalized Newton method to Riemannian manifolds with the exponential mapping. However, the exponential mapping is often computationally expensive. Therefore, in this thesis, we propose the Riemannian generalized Newton method with the concept of retractions, which is a generalization of the exponential mapping with generally less computational cost. To this end, we define Riemannian counterparts of the concepts such as semismooth vector fields and SC^1 functions in Euclidean spaces, and we prove that the proposed concepts on Riemannian manifolds have properties similar to the Euclidean ones. Then, we prove that the proposed algorithm has the same convergence property as the algorithm with the exponential mapping.

In addition, we show that the proposed method can be applied to constrained optimization problems on Riemannian manifolds, which have been attracting attention in recent years, by using the augmented Lagrangian method, which involves SC^1 optimization in its subproblems. Furthermore, we verify the validity of the proposed method from the numerical results on solving the absolute value vector field problem and nonnegative PCA problems, where the proposed algorithm is used to find a zero of a semismooth vector field and to minimize an SC^1 function, respectively. We show that the proposed method with a retraction is faster than the generalized Newton method with the exponential mapping and converges with a considerably less number of iterations than the steepest descent method.