Master's Thesis

An Uncertainty Model for Positive-Valued Parameters with Application to Robust Optimization

Guidance

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Abstract

In most of the real-world applications of mathematical optimization, we do not know the precise value of the parameters or the data. Robust optimization is one of the modeling methodologies that deals with such uncertainties. It assumes that the uncertain parameters belong to a set called uncertainty set, and implements the optimization according to the worst situation among the set. There are a number of areas where robust optimization is applied, including, for example, truss topology design, portfolio selection, and machine learning. As models of uncertainty, it is common to use boxes or ellipses to define the uncertainty sets. Since robust optimization problems are generally semi-infinite programming, they are not always numerically tractable. One way to deal with this problem is to reformulate the robust optimization problem to a tractable form using Fenchel duality by considering the assumptions on the original problem and the uncertainty set.

In this paper, we propose a new model of uncertainty set that aims to handle optimization problems with only positive-valued parameters. This model uses a particular convex function that measures the variation of uncertain parameters from their nominal values. We derive a tractable equivalent form of the robust problem with our uncertainty model by using duality techniques. In addition, we show some properties of robust optimization problems using the proposed model as an uncertainty set. These properties confirm that our model can keep the "positivity" of the assumed worst-case value of uncertain parameters, and they can be used to determine the size of the uncertainty set before solving the robust problem. Moreover, we derive a probabilistic guarantee result for the robust feasible solutions to remain feasible with respect to realized scenarios of the uncertain parameter with lognormal distributional assumption. In the numerical experiments, we solved two application problems with positive-valued parameters, applying our uncertainty model. For these problems, traditional uncertainty models may lead to infeasibility if we consider their robust counterpart. We confirmed the tractability of the robust counterpart of such problems in the case where the proposed model is used. Moreover, we observed that the robust solutions not only can reduce the violation of the constraints but also can outperform the nominal solutions in terms of risk-adjusted objective function value.