Master's Thesis

A New Quasi-Newton Method for Unconstrained Multiobjective Optimization Problems

Guidance

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Abstract

In multiobjective optimization, we consider problems with several objective functions and aim to minimize these functions simultaneously. In recent years, extensions of single objective descent methods, such as the steepest descent, the Newton, and the quasi-Newton methods have been developed, all of them having theoretical guarantee of convergence. In the steepest descent method, we can obtain stationary points regardless of the convexity of objective functions, but the rate of convergence is linear. On the other hand, the sequence of points generated by the Newton method converges superlinearly. However, we have to assume the Hessians of the objective functions to be positive definite. A method that lies between these two approaches is the quasi-Newton, but the one proposed for multiobjective problems needs to solve quadratically constrained quadratic programming subproblems, which can be computationally expensive.

In this paper, we propose a new quasi-Newton method which has convergence rate close to the Newton method and can solve nonconvex problems. Differently from the existing multiobjective quasi-Newton method, the subproblems considered here are easier to solve, with only linear constraints. Additionally, we solve the dual of such subproblems in order to reduce the computational cost for problems with large variable dimensions. We also update the approximations of the Hessians using the solutions of the dual subproblems, together with a modified BFGS formula. Finally, we confirm the effectiveness of our algorithm with some numerical experiments.