Master's Thesis

A generalized restart FISTA with Bregman distance for convex optimization

Guidance

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Abstract

Over the past decade or so, first-order methods for convex optimization problems have been the subject of active study due to their usefulness in addressing large-scale problems that arise in fields such as statistics, machine learning, and others. These methods are iterative, in the sense that they generate sequences of approximation solutions by utilizing information of first-order differentials, such as the gradients or subgradients of the objective functions. Notable examples of first-order methods include the steepest descent method, subgradient method, and proximal gradient method. Recently, the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA), which is valid for convex optimization problems with a specific problem structure, and restart FISTA, which mitigates the drawbacks of FISTA, have been developed. However, one drawback of both FISTA and restart FISTA is that the regularization term of the subproblem is fixed at each iteration, making it a bad approximation of the original problem, and consequently increasing the overall number of iterations.

In this paper, to overcome the above drawbacks, we propose a restart FISTA algorithm that uses the Bregman distance as the regularization term of the subproblem and modifies the Bregman distance at each restart. The proposed method is anticipated to converge faster by bringing the subproblem closer to the original problem at each restart while maintaining the computational time for solving the subproblem. Also, as an example of the restart FISTA that changes the regularization term of the subproblem at each iteration, we propose a variant for the ℓ_1 regularization problem. Specifically, we change L, which plays the role of stepsize in FISTA, at each restart and use the backtracking strategy to search for L that guarantees the convergence of FISTA. The proposed method guarantees global convergence by making appropriate assumptions on the Bregman distance. Finally, the validity of the proposed method is confirmed by several numerical experiments on the ℓ_1-ℓ_2 regularization problem.