Master's Thesis

A projected gradient method for vector optimization with positive semidefinite cone and its application to experiment design

Guidance

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Abstract

Vector optimization consists in optimization problems where multiple objective functions are optimized with respect to a partial order induced by a cone. It generalizes the multiobjective optimization, where the given cone is the nonnegative orthant. For vector optimization problems with easy constraints and vector-valued objective function, one can use the projected gradient method. In this work, we study this method, focusing on vector optimization problems with matrix-valued objectives, where the partial order is induced by the semidefinite cone. Our extended projected gradient method is similar to the original one, but involves distinct subproblems, which are necessary to determine search directions. To set up the subproblem, we require the specification of a generator of the semidefinite cone. By selecting an appropriate generator, the subproblem can be written as a minimization problem involving the maximum eigenvalue of a matrix, leading to a linear semidefinite programming formulation. We note that this generator differs from those used in the previous works. Based on this, we prove that all accumulation points of the sequence generated by the proposed method, if they exist, converge to Pareto stationary points. Moreover, for the numerical experiments, we consider the so-called experiment design problem, which consists in determining the most informative approach for conducting experiments. Classical methods for this problem include the scalarization methods called D-optimal, E-optimal, and A-optimal designs. We then perform numerical experiments utilizing these scalarizations, as well as the proposed method, and an approach called Pascoletti-Serafini scalarization for comparative analysis. We show that the proposed method can find distinct Pareto solutions that cannot be found using scalarization techniques, which can be seen as an advantage from the decision-making perspective.