Master's Thesis

Multi-objective accelerated proximal gradient methods with restart techniques

Guidance

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Abstract

The multi-objective optimization (MO) is a problem that finds Pareto optima of multiple objective functions. In this thesis, we focus on unconstrained multi-objective minimization problems, where each objective function is expressed as the sum of a continuously differentiable function and a closed, proper, and convex function. The existing algorithms for this problem include the proximal gradient method (PGM) and its accelerated version called the accelerated proximal gradient method (APGM). The PGM has been shown to converge in $O(1/k)$ for convex objective functions and $O(\sqrt{1/k})$ for nonconvex ones. The APGM, which requires the convexity of all objective function, has the convergence rate of $O(1/k^2)$, which is faster than the PGM. However, the sequence of the objective functions' values generated by the APGM is not necessarily monotonically decreasing, which might cause slow convergence for some problems. To overcome the drawback, the monotone APGM and the restart APGM have been proposed for single-objective optimization problems. Recently the monotone APGM was extended to MO, but it is not computationally efficient. In this thesis, we propose a restart APGM for MO. The proposed method restarts from the current iterate when its progress is stagnant. We establish that the convergence rate of the proposed method to a Pareto stationary is $O(1/k^2)$ for convex and $O(\sqrt{1/k})$ for non-convex problems. To the best of the author's knowledge, this is the first complexity analysis of APG type methods for the non-convex MO. We conducted several numerical experiments, which show that the proposed method is competitive to the existing methods.