Master's Thesis

The Utilization of Global Optimization for Obtaining Generalized Nash Equilibria

Guidance

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Abstract

In this thesis, we focus on the generalized Nash equilibrium problem (GNEP) through the lens of global optimization. GNEP, pivotal in various fields such as economics and sociology, is an NP-hard problem. The most popular approach to solving GNEP is to reformulate it into an optimization problem, and solve it via a tailored optimization method. There are two major ways in the reformulation: to utilize the Nikaido-Isoda function, or to leverage the quasi-variational inequality. Most of the research available mainly focus on solving GNEP with concave utility functions and related computational complexity. However, even when GNEP is reformulated into a rather simple optimization problem, it is a non-convex optimization problem, and hence is difficult to obtain a generalized Nash equilibrium (GNE). Moreover, when a utility function is not concave, a global optimum of the reformulated optimization problem is not necessarily a GNE. Therefore, we need a procedure that checks whether the global optimum is exactly a GNE. In our work, we first propose a procedure to solve non-convex GNEP. It reformulates the GNEP into the generalized variational inequality, and then further reformulate it to an optimization problem of the regularized gap function. The regularized gap function is evaluated by solving a certain quadratic optimization problem, which is time consuming and causes the difficulty to apply the population based global optimization such as the genetic algorithm. To overcome the difficulty, we exploit the duality of the quadratic optimization. We then formulate the equivalent optimization problem whose objective function is easy to evaluate. Next, we propose a procedure for checking whether the global optimum is exactly a GNE. Finally, we present some numerical results that illustrate the effectiveness of our procedures.