Master's Thesis

Block Coordinate Descent Network Simplex Methods for Solving Large-Scale Optimal Transport Problems

Guidance

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Abstract

Optimal Transport (OT) problems provide a mathematical framework for transporting resources while minimizing costs and play a fundamental role in various fields, including machine learning, economics, and biology. Traditional algorithms for solving OT problems include the Sinkhorn algorithm and the Network Simplex (NS) method. The Sinkhorn algorithm efficiently approximates solutions using entropy regularization but lacks the ability to compute exact solutions. The NS method, despite its capability of obtaining exact solutions, suffers from slow convergence when applied to large-scale problems.

To overcome these limitations, we propose the Block Coordinate Descent Network Simplex (BCDNS) method, which integrates the Block Coordinate Descent (BCD) method with the NS method. By decomposing the OT problem into smaller subproblems, BCDNS significantly reduces the computational burden. A key technique of our method is the selection of basis variables from previous subproblems to ensure feasibility and accelerate convergence. The proposed method retains the exact solution property of the NS method while achieving faster convergence. Moreover, by solving smaller subproblems at each iteration, the method reduces memory requirements compared to Sinkhorn algorithm. We show that the proposed BCDNS method finds an exact optimal solution within a finite number of iterations. Then we report some numerical experiments. The numerical results demonstrate that BCDNS outperforms the NS methods in terms of runtime efficiency. Furthermore, it is also faster than the Sinkhorn algorithm when we require a highly accurate solution.