Master's Thesis

An Augmented Lagrangian Method for Worst-Case Conditional VaR Optimization with Asset Constraints

Guidance

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Abstract

In this thesis we consider a practical use of the Worst-Case Conditional Value at Risk (WCVaR) in portfolio optimization. The WCVaR is a risk measure defined as the worst value of the Conditional Value at Risk (CVaR) evaluated within an uncertain distribution set. Although the WCVaR is a more realistic risk measure than the CVaR, its evaluation is expensive. Moreover, the WCVaR optimization with a cardinality constraint has not been considered yet. The cardinality-constraint is a restriction on the number of assets invested, and it is a common requirement in practical investment.

To address them, we propose a WCVaR model with the cardinality-constraints. When the WCVaR is formulated with a linear risk function, the proposed model can be reformulated as a mixed integer programming problem (MIP), and hence it is solved by the state-of-the-art MIP solvers such as Gurobi. When the model is large or the risk function is nonlinear, it is difficult to get an exact solution by the MIP solvers. Therefore, we also develop an augmented Lagrangian algorithm to get a practical solution within a reasonable time. To this end, we convert the cardinality-constraints with some relaxed constraints proposed by Kanzow, Raharja, and Schwartz. Since the relaxed constraints consist of differentiable functions only, the converted optimization problem becomes a differentiable optimization problem, and hence it can be solved by the augmented Lagrangian method. We propose an efficient implementation of the augmented Lagrangian method by exploiting a special structure of WCVaR and the Moreau envelope. We conducted numerical experiments with real stock market data. We solved both of the MIP formulation and the converted differentiable optimization problem by Gurobi and the proposed augmented Lagrangian method, respectively. The results show that we can get a practical portfolio within a reasonable time.