NAME:

DEPARTMENT:

OR Advanced, Part 1, Small Report No. 1

- ✓ You can use your annotations, books and internet for this report. You can also consult this class' professor for some hint. Just be sure to write your own answer.
- ✓ Answer the questions in Japanese or English.
- ✓ We will have two small exercise-type reports (October 17 and 31) and one final report. The final one corresponds to 70% of the grade of the "nonlinear optimization" part, while each small report corresponds to 15% of this total grade.
- ✓ For this small report, if you submit it in the end of this class (October 17), the total possible grade for it is 15.
- ✓ If you were not able to finish it during the class, or if you were absent, you may submit it in the beginning of the next class (October 24). However, the total possible grade for it will be 10 instead of 15.

Consider the following nonlinear programming problem:

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_i(x) \leq 0, \quad i = 1, \ldots, m, \end{array} \tag{NLP}$$

where $f\colon \mathbb{R}^n \to \mathbb{R}$ and $g\colon \mathbb{R}^n \to \mathbb{R}^m$ are continuously differentiable and convex functions. Assume that $(x^*, \lambda^*) \in \mathbb{R}^n \times \mathbb{R}^m$ satisfies the KKT conditions.

Exercise 1. Prove that the function $L\colon \mathbb{R}^n \to \mathbb{R}$ defined below is convex:

$$L(x) := f(x) + \sum_{i=1}^{m} \lambda_i^* g_i(x).$$

Exercise 2. Show that x^* is a global minimizer of L.

Exercise 3. Prove that x^* is a global minimizer of (NLP).